The Challenge of Negative Refraction

JB Pendry
The Blackett Laboratory, Imperial College London
http://www.cmth.ph.ic.ac.uk/photonics/

Recent Reviews of Negative Refraction

Negative Refraction
Contemporary Physics **45** 191-202 (2004)
JB Pendry

Reversing Light with Negative Refraction
Physics Today **57** [6] 37-43 (June 2004)
JB Pendry and DR Smith

Metamaterials and Negative Refractive Index
Science **305** 788-92 (2004)
DR Smith, JB Pendry, MCK Wiltshire

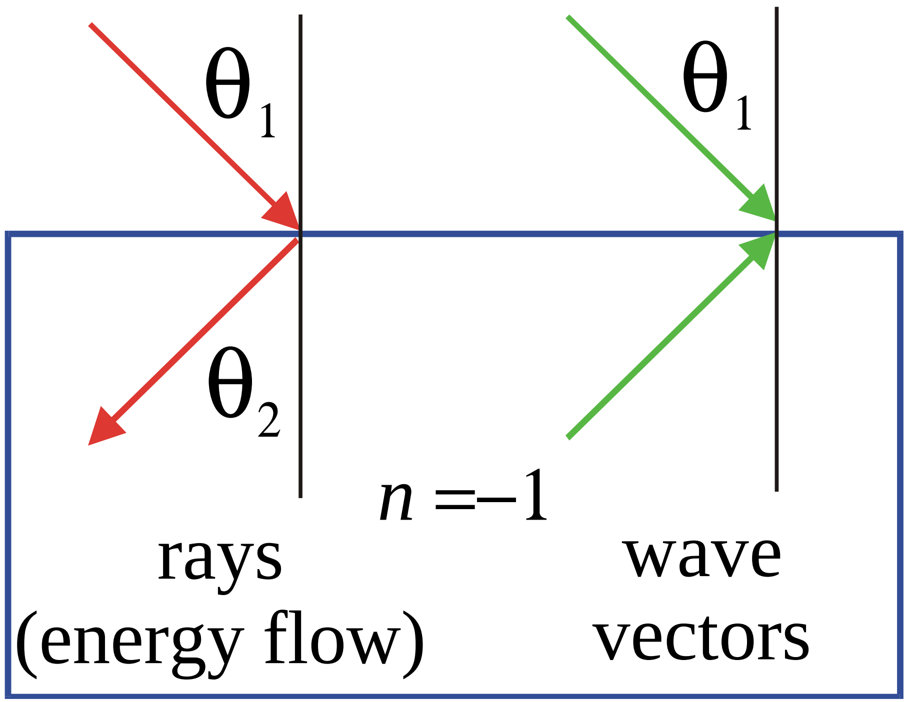
Negative Refraction - State of the Theory

Veselago negative refraction	– 1968
Perfect focussing	– 2000
Negative space	– 2003
Plastic space	– 2003

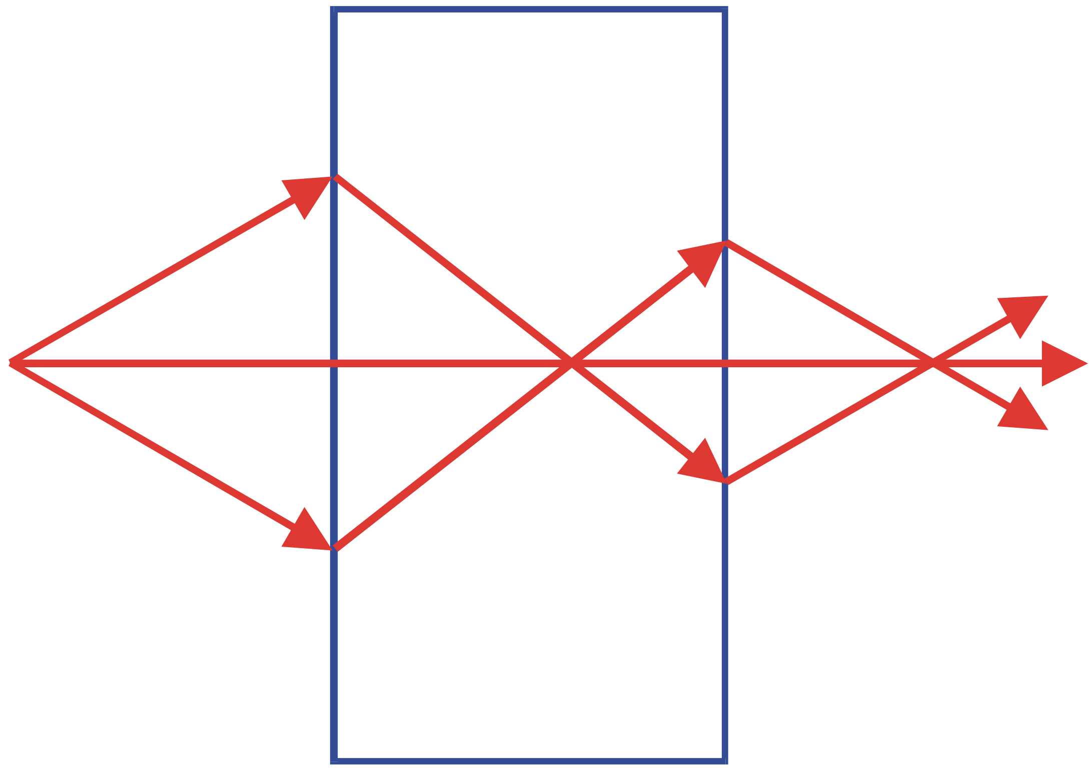
Negative Refractive Index and Snell's Law

$$n = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$

Hence in a negative refractive index material, *light makes a negative angle with the normal*. Note that the parallel component of wave vector is always preserved in transmission, but that energy flow is opposite to the wave vector.



Negative Refractive Index and Focussing



A negative refractive index medium bends light to a negative angle relative to the surface normal. Light formerly diverging from a point source is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time.

The consequences of negative refraction
3. *Perfect* Focussing

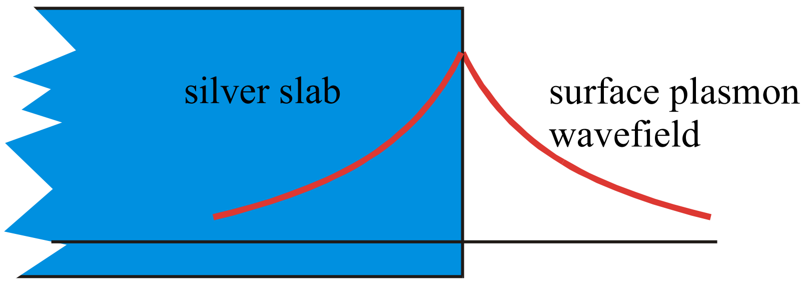
A conventional lens has resolution limited by the wavelength. The missing information resides in the near fields which are strongly localised near the object and cannot be focussed in the normal way.

The new lens based on negative refraction has *unlimited resolution* provided that the condition $n = -1$ is met exactly. This can happen only at one frequency. (Pendry 2000).

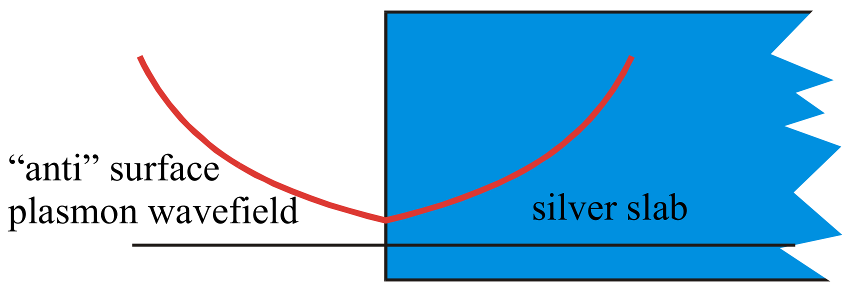
The secret of the new lens is that it can focus the near field and to do this it must *amplify* the highly localised near field to reproduce the correct amplitude at the image.

Anatomy of a Superlens

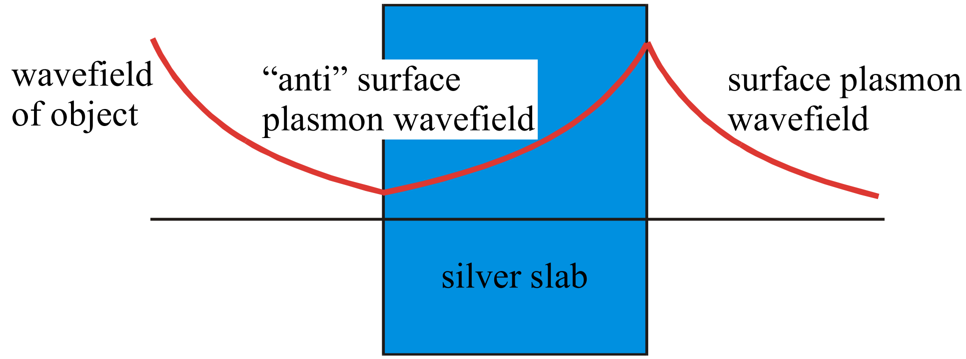
The superlens works by **resonant excitation of surface plasmons in the silver**,



At the same frequency as the surface plasmon there exists an unphysical "anti" surface plasmon - wrong boundary conditions at infinity,



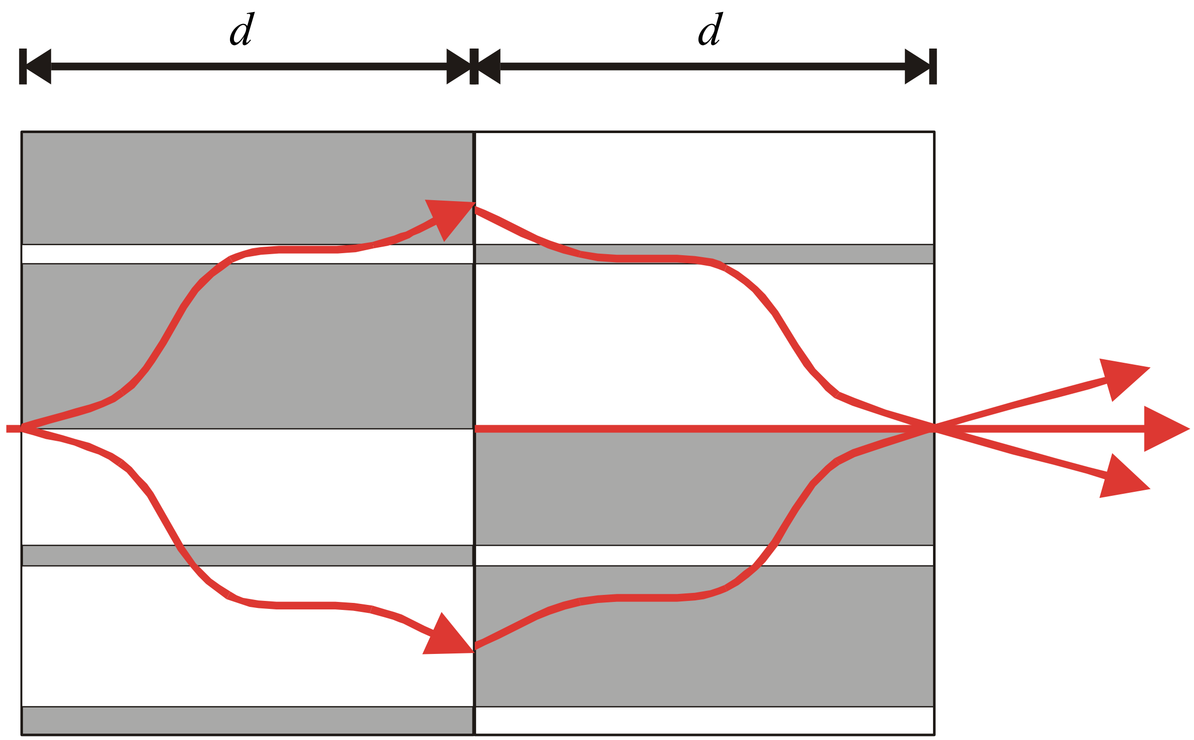
However,



Matching the fields at the boundaries selectively excites a surface plasmon on the far surface.

Negative Space

A slab of $n = -1$ material thickness d, cancels the effect of an equivalent thickness of free space. i.e. objects are focussed a distance $2d$ away. An alternative pair of complementary media, each cancelling the effect of the other. The light does not necessarily follow a straight line path in each medium:

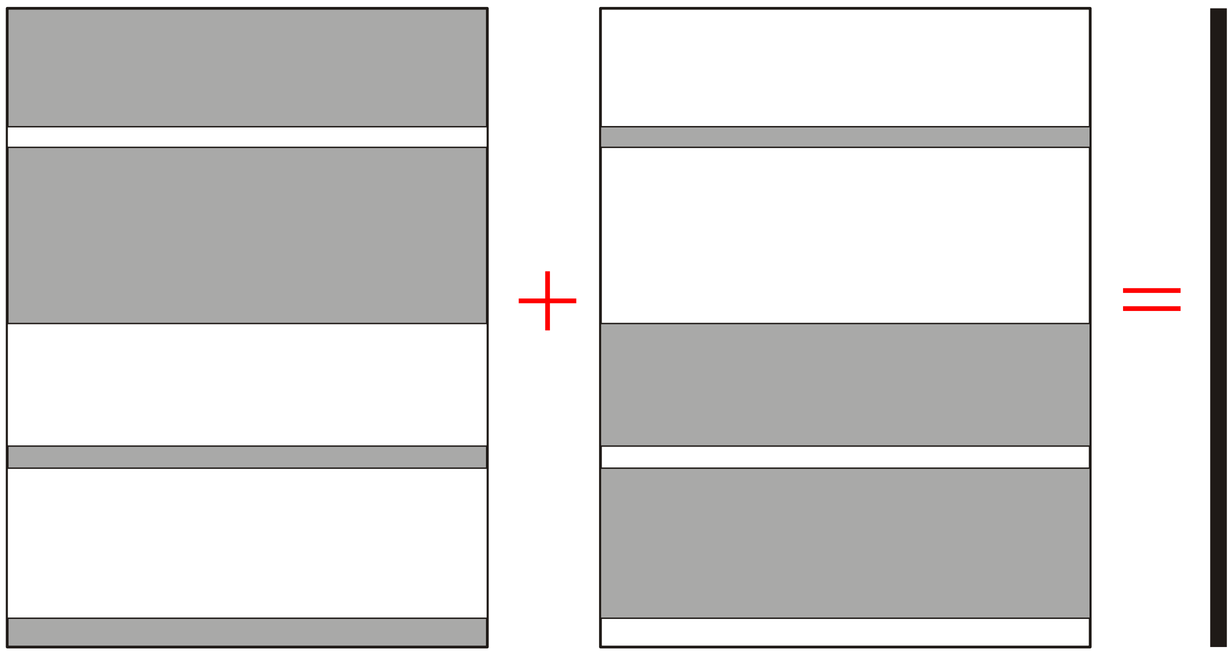


General rule:
two regions of space optically cancel if in each region ε,μ are reversed mirror images.

The overall effect is as if a section of space thickness $2d$ were removed from the experiment.

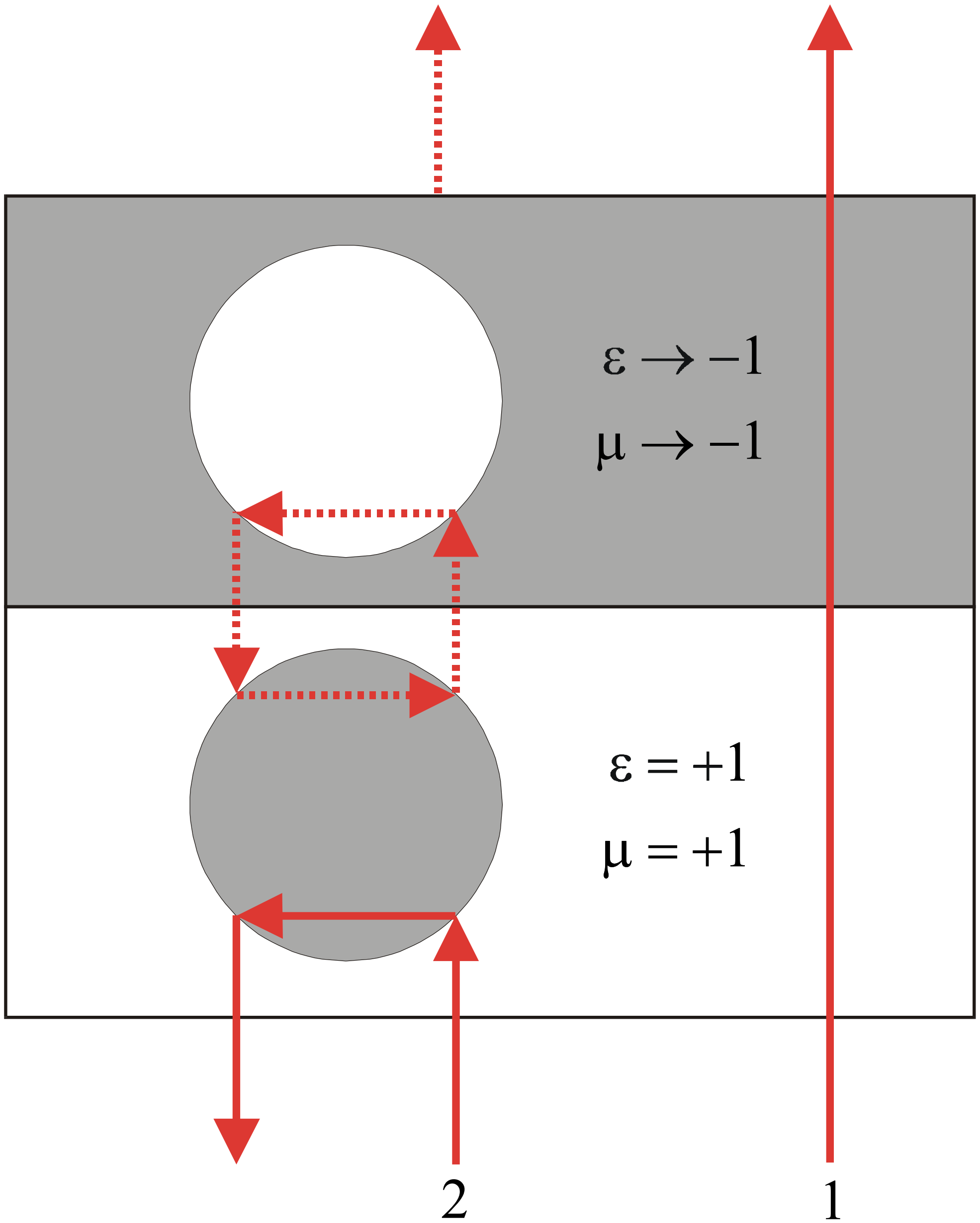
Complementary Media

We can express our theorem in a graphical fashion: two complementary media have an optical sum of zero. We calculate the optical response of the rest of the system outside the lens region by cutting out the complementary media which comprise the lens and closing the gap between the two remaining halves of the systems.



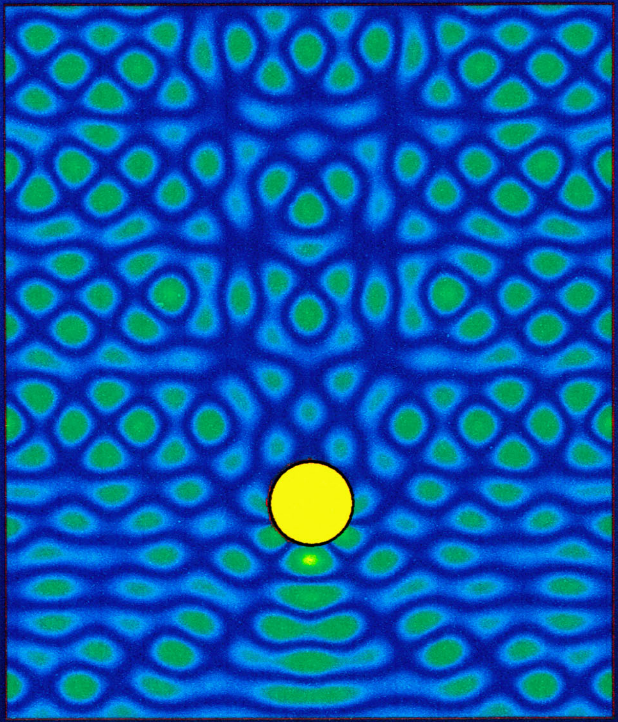
A graphical expression of our new theorem: complementary halves sum to zero. **The optical properties of the rest of the system can be calculated by cutting out the media and closing the gap.**

A Negative Paradox

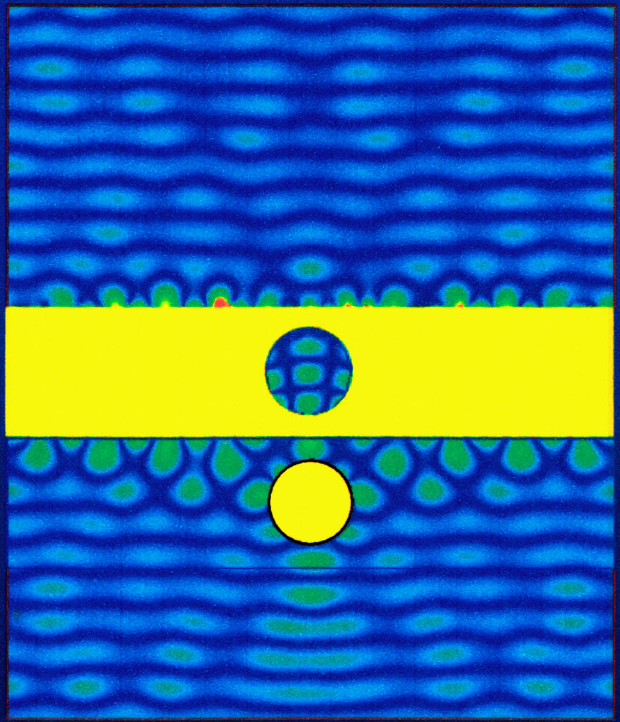


The left and right media in this 2D system are negative mirror images and therefore optically annihilate one another. However a ray construction appears to contradict this result. Nevertheless the theorem is correct and the ray construction erroneous. Note the closed loop of rays indicating the presence of resonances.

Compensation of inhomogeneous media



Scattering from a cylinder with n=-1.4



Compensation of the n=-1.4 cylinder

Change the Shape: Change ε, μ

see A J Ward & J B Pendry *Journal of Modern Optics*, **43** 773-93 (1996).

step 1: make a coordinate transformation,

$$q_1(x,y,z), \quad q_2(x,y,z), \quad q_3(x,y,z)$$

step 2: rewrite Maxwell's equations in the new coordinate system – the *form* of Maxwell's equations does not change,

$$\nabla\times\tilde{\mathbf{E}} = -\tilde{\mu}\mu_0\partial\tilde{\mathbf{H}}/\partial t, \quad \nabla\times\tilde{\mathbf{H}} = -\tilde{\varepsilon}\varepsilon_0\partial\tilde{\mathbf{E}}/\partial t$$

step 3: Calculate the new values of ε, μ,

$$\tilde{\varepsilon}_i = \varepsilon_i\frac{Q_1Q_2Q_3}{Q_i^2}, \quad \tilde{\mu}_i = \mu_i\frac{Q_1Q_2Q_3}{Q_i^2}$$

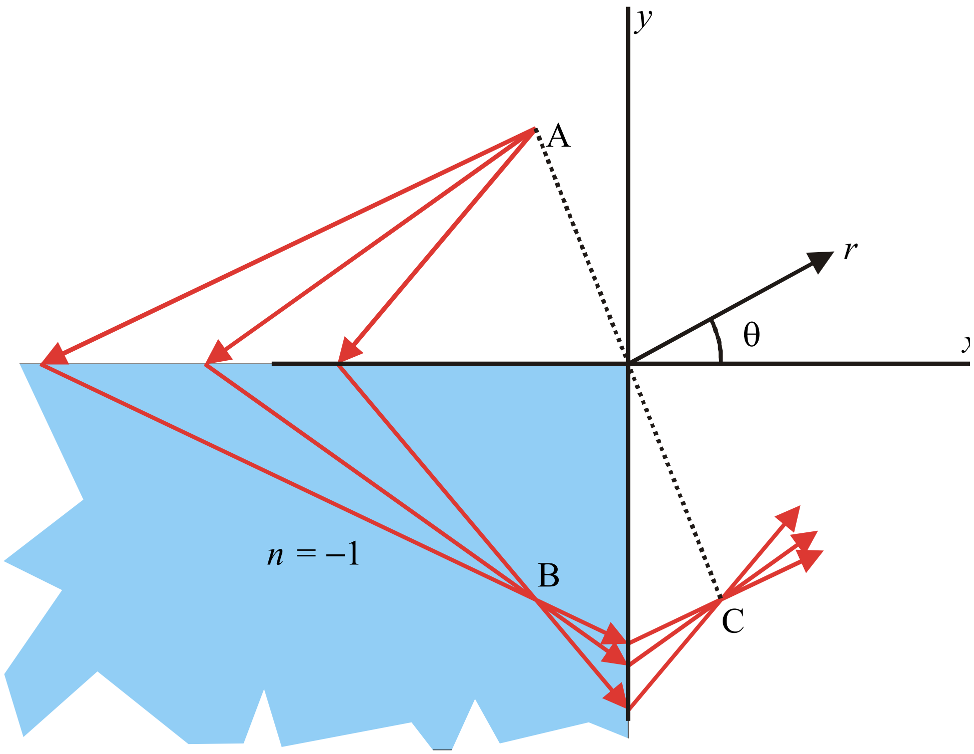
where,

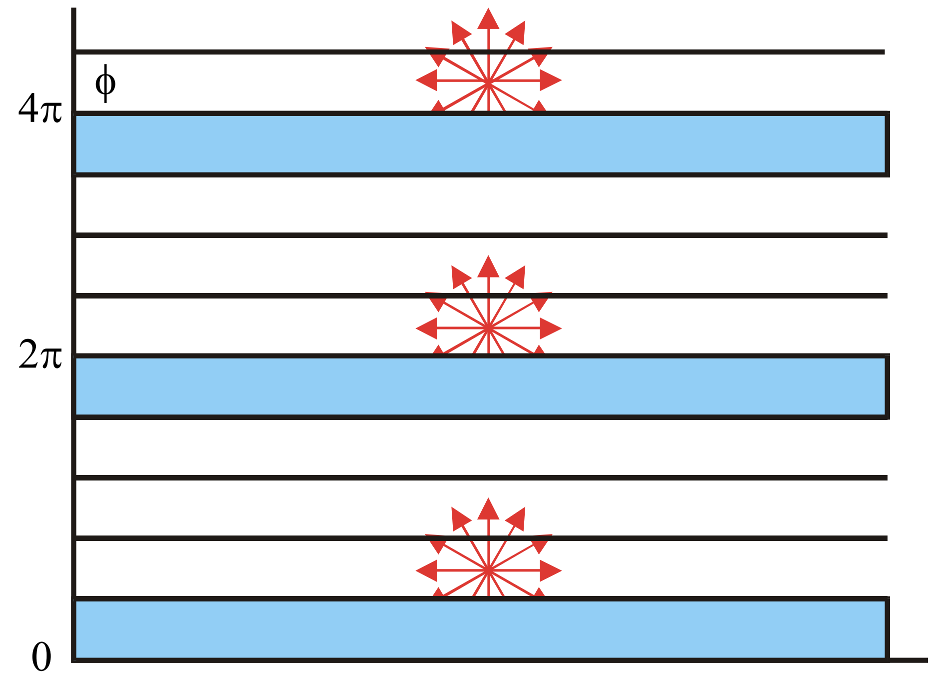
$$Q_i^2 = \left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2, \quad \tilde{E}_i = Q_iE_i, \quad \tilde{H}_i = Q_iH_i$$

conclusion:

We can transform the perfect lens solution into new geometries where the surfaces are curved, but we must change ε, μ.

‘Perfect’ Corner Reflectors by Negative Refraction

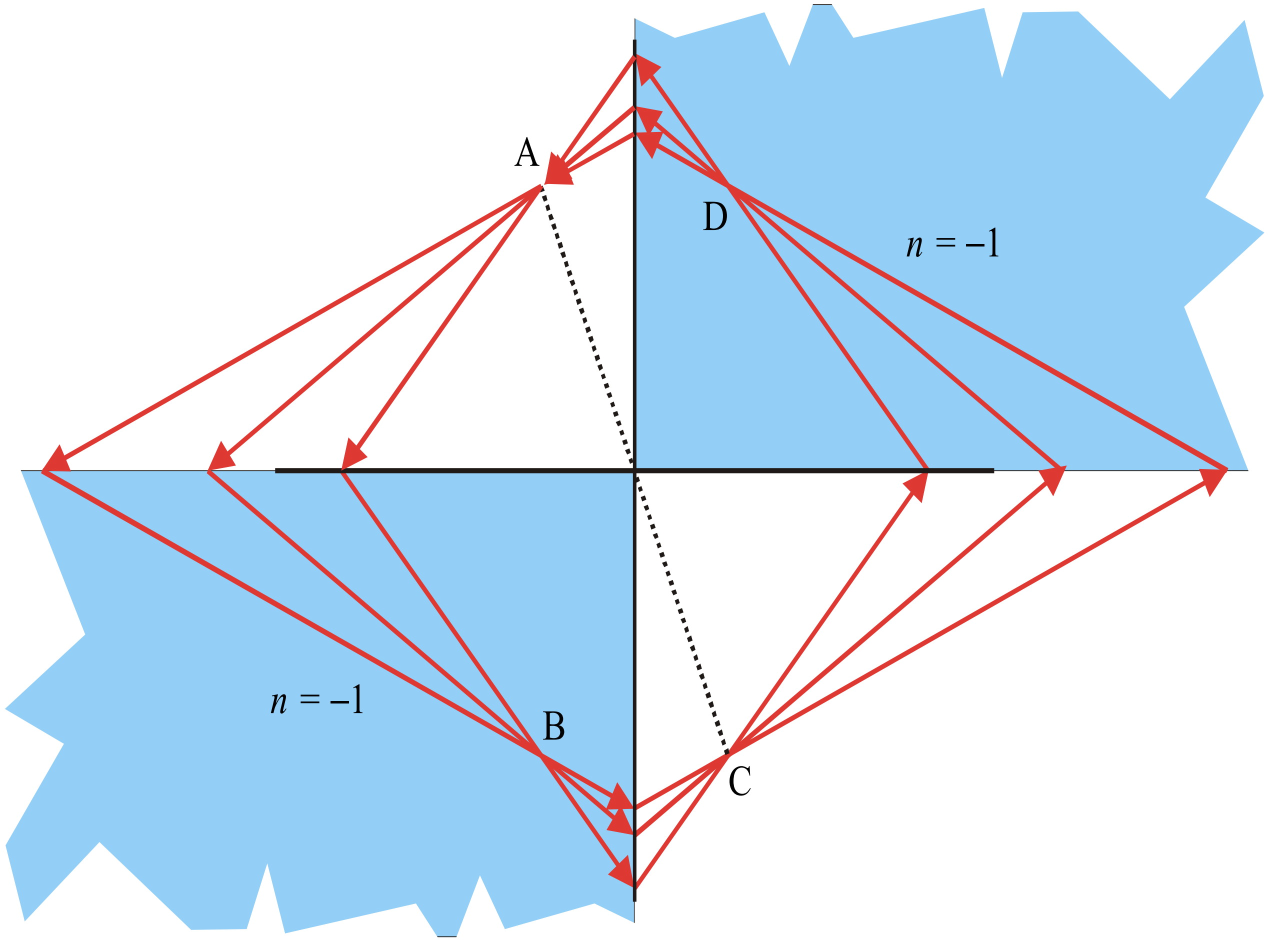




A negatively refracting corner reflector. The direction of each incident wave vector is reversed. and appears to radiate from point C, rotated by 180º about the corner from the true origin at A. The images at B and C include the correct near field components.

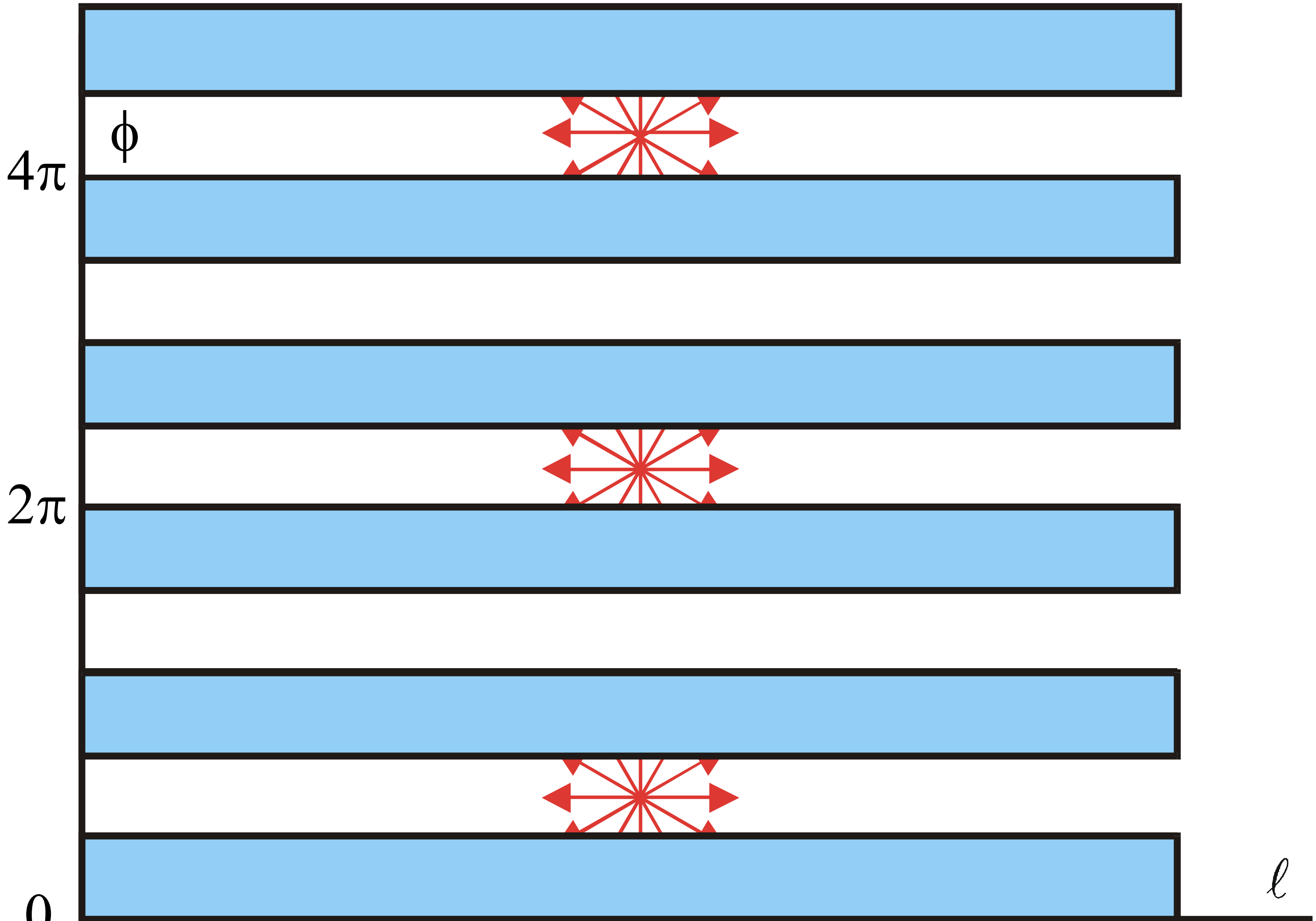
A coordinate transformation maps ϕ into the new y-axis, and r into the new x-axis. Hence the corner cube is equivalent to a stack of slabs, where every fourth slab in the stack is complementary to the other 3.

Two Negative Corners



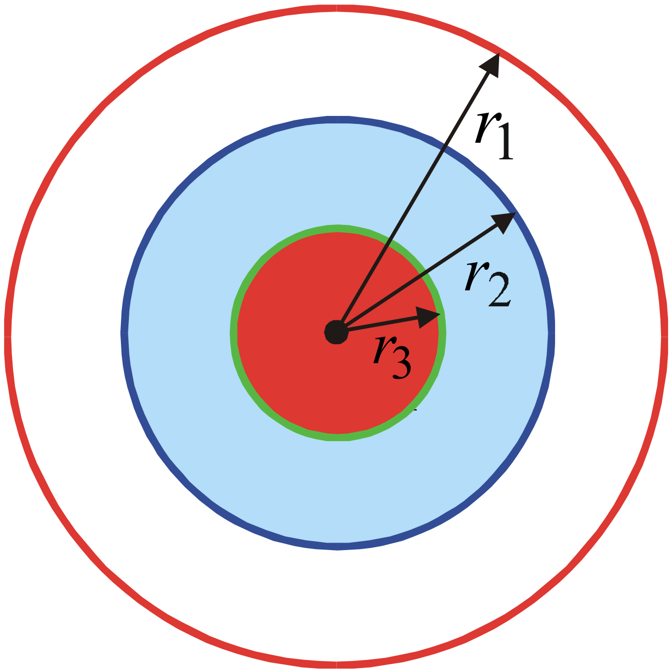
Notomi realised that a double negative corner can be constructed from $n = -1$ material. This configuration *cannot* be accomplished with mirrors. After focussing at B, C, D, the rays return to the source point, A, with the same phase with which they started out. However this configuration is highly singular because the two spaces are self-annihilating. The fields grow without limit as a function of time.

Two Negative Corners – Perfect!



The previous figure can be mapped into a stack of slabs, where every succeeding slab in the stack is complementary to the previous one. Note that this space is self-annihilating and the multiple sources appear to lie on top of on another.

A Perfect Magnifying Glass



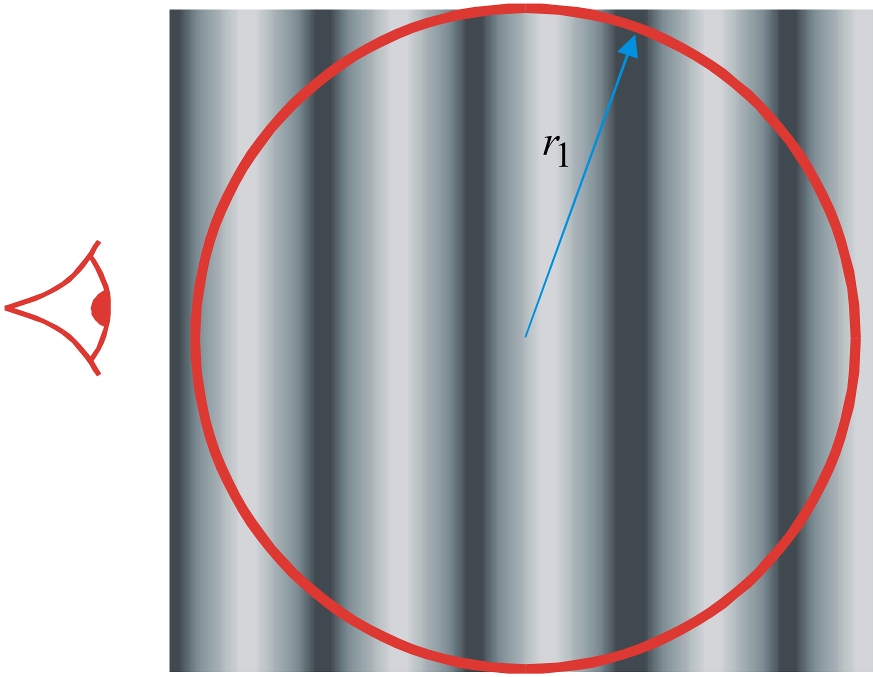
$$\varepsilon_x = \varepsilon_y = \varepsilon_z = +\frac{r_2^2}{r_3^2}, \qquad 0 < r < r_3$$

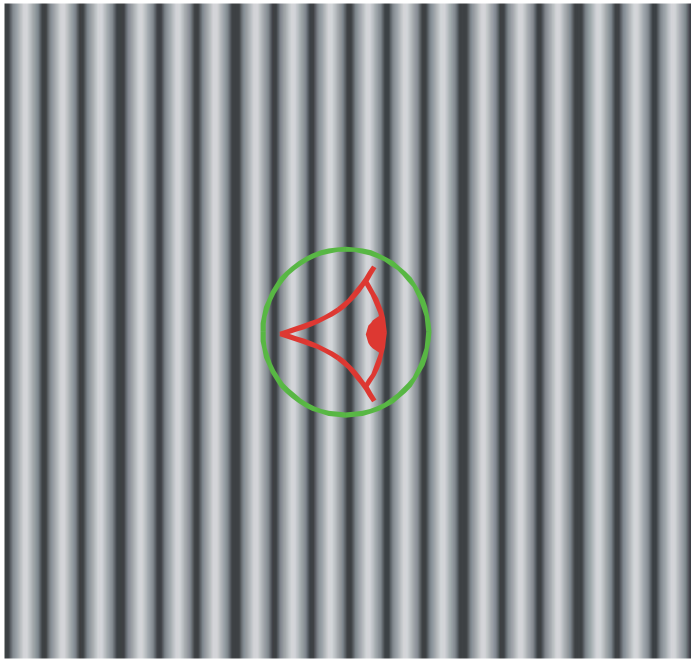
$$\varepsilon_x = \varepsilon_y = \varepsilon_z \to -\frac{r_2^2}{r^2}, \qquad r_3 < r < r_2$$

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = +1, \qquad r_2 < r < \infty$$

$$\mu_x = \varepsilon_x, \quad \mu_y = \varepsilon_y, \quad \mu_z = \varepsilon_z$$

It is possible to design a spherical annulus of negative material lying between r_2 and r_3 that acts like a magnifying glass. To the outside world the contents of the sphere radius r_3 appear to fill the larger sphere radius r_1 with proportionate magnification.





Our design objective. Left, to observers external to the red circle, $r > r_1$, the system is invisible and appears to be transparent to incident radiation. Right: to observers internal to the green circle, $r < r_3$, there is also no evidence of the boundary at $r = r_3$ which is perfectly transparent to outgoing radiation. The material between r_1 and r_3 acts as a wavelength expander/contractor

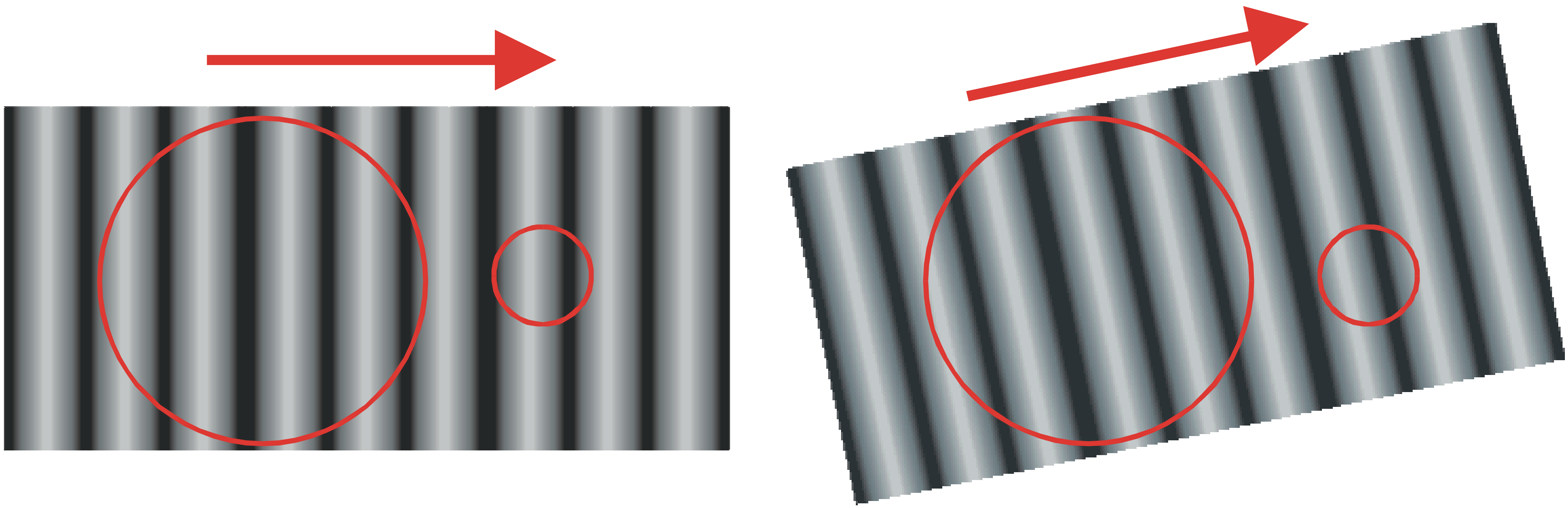
$$\begin{array}{llll} \varepsilon_x = +1, & \varepsilon_y = +1, & \varepsilon_z = +1, & r_2 < r \\ \varepsilon_x = -1, & \varepsilon_y = -1, & \varepsilon_z = -r_2^4 / r^4, & r_3 < r < r_2 \\ \varepsilon_x = +1, & \varepsilon_y = +1, & \varepsilon_z = +r_2^4 / r_3^4 = +r_1^2 / r_3^2, & r < r_3 \end{array}$$

and with identical values for,

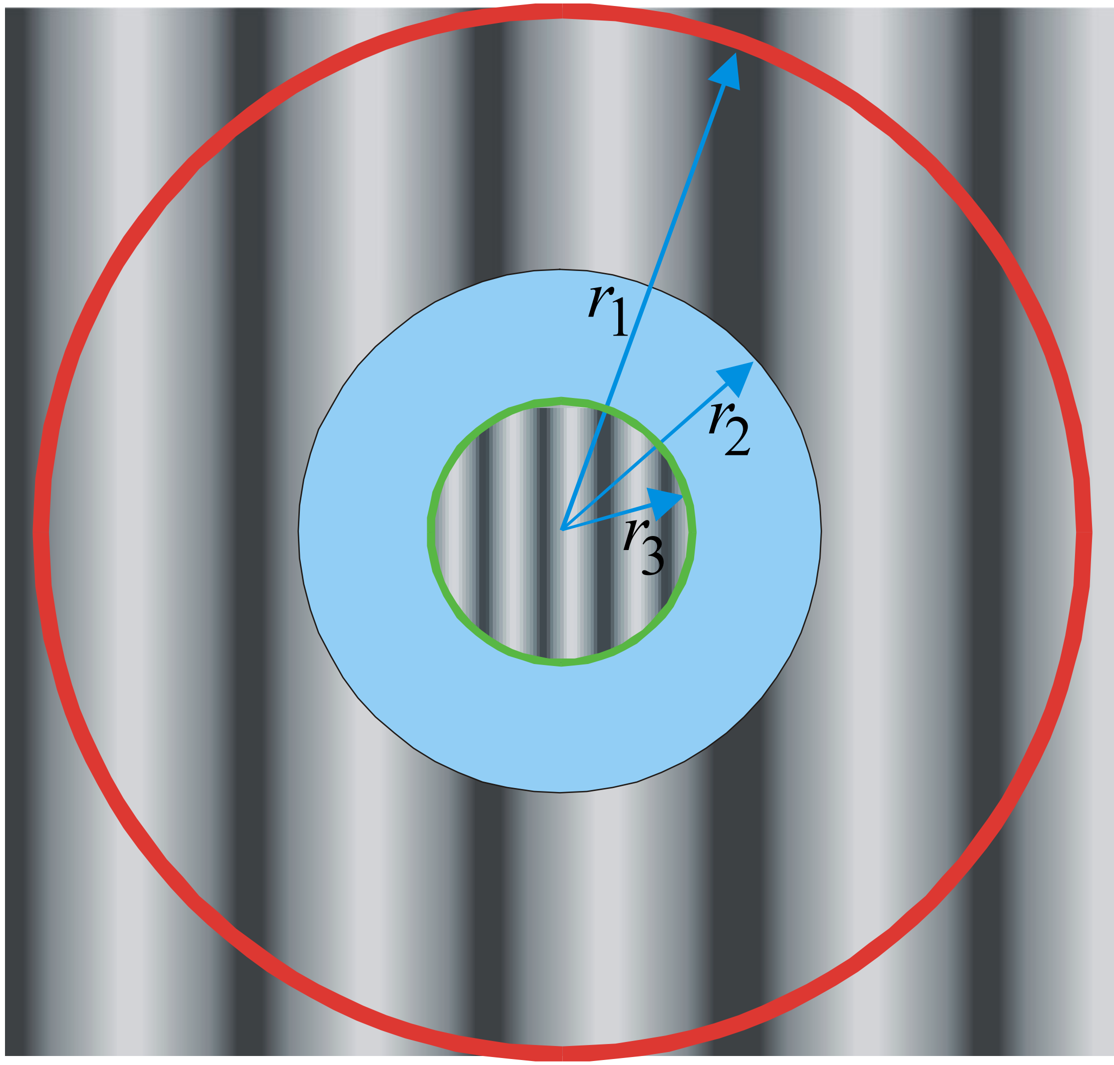
$$\mu_x = \varepsilon_x, \quad \mu_y = \varepsilon_y, \quad \mu_z = \varepsilon_z$$

Imperial College London

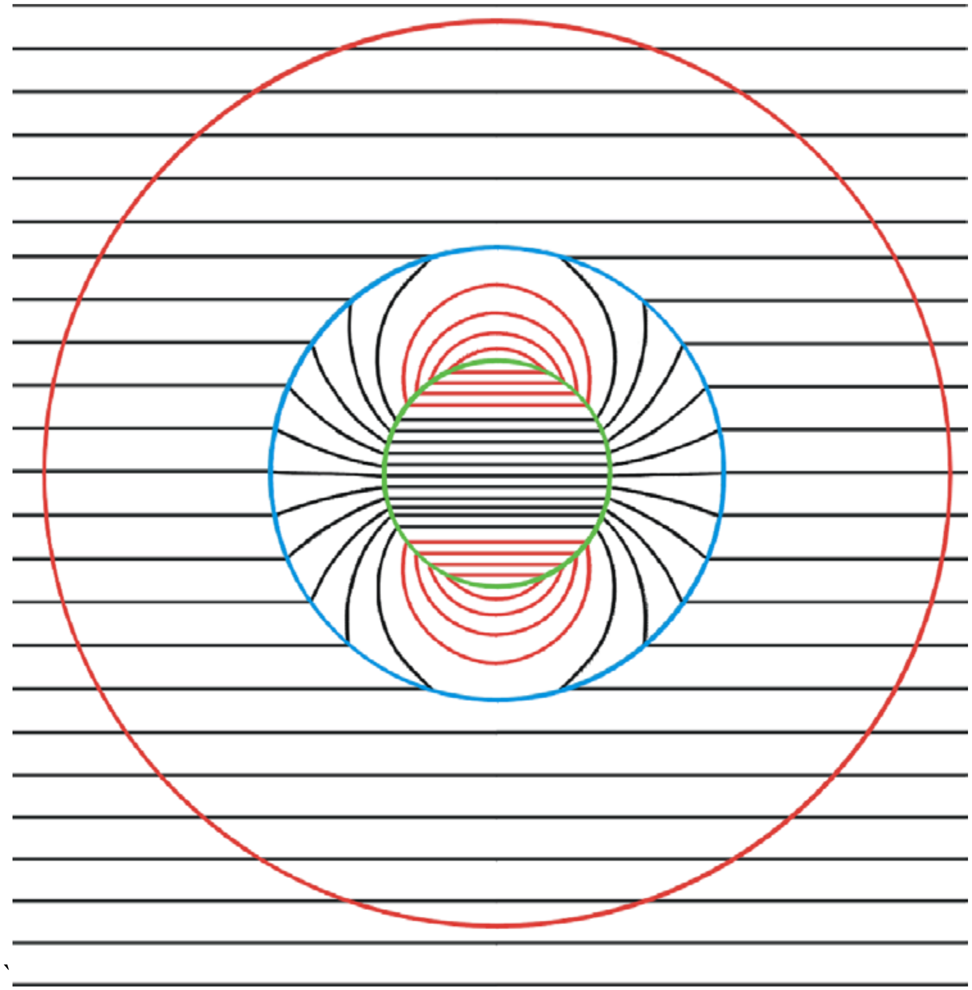
Can we beat the aperture limit for angular resolution?



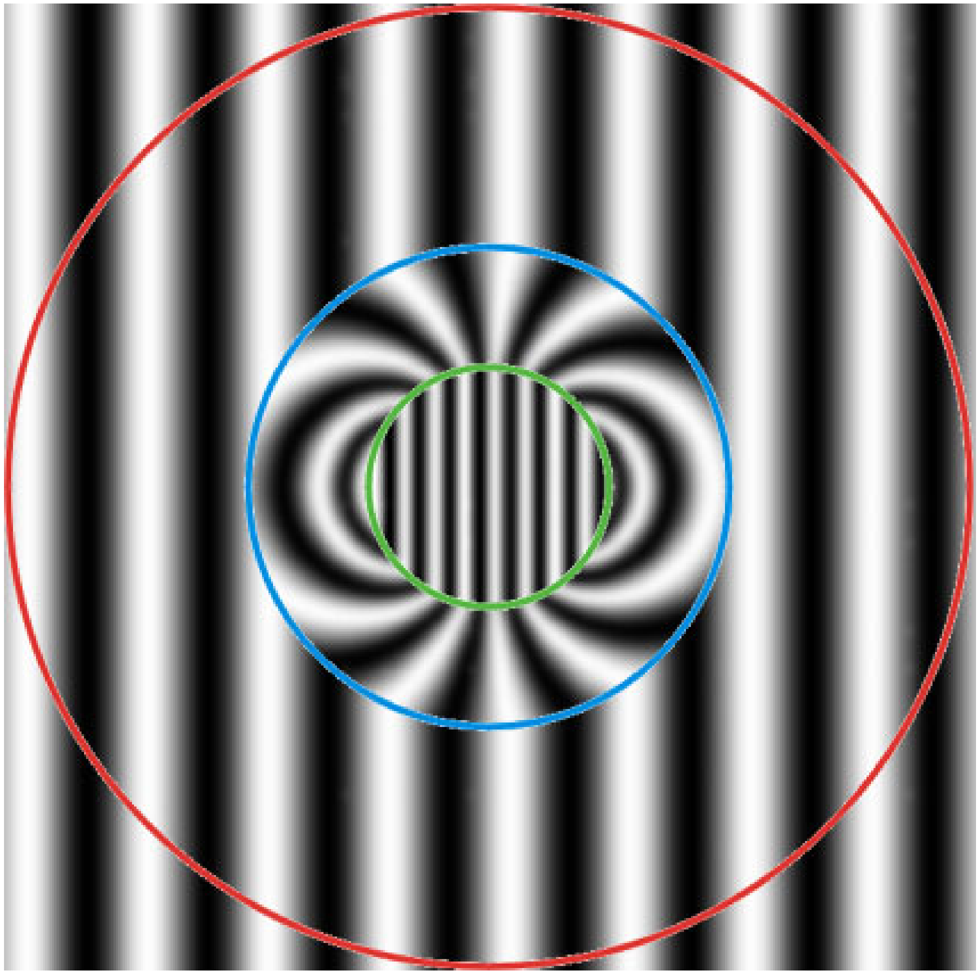
Devices detect the direction of a wave by the oscillations on the surface of the detector. Obviously a larger detector senses more oscillations and is therefore more sensitive to direction.



A suitably designed negative material (blue shading) placed in the cylindrical annulus between r_2 and r_3 will compress the wave field originally within the cylinder r_1 to fit inside the smallest cylinder radius r_3.

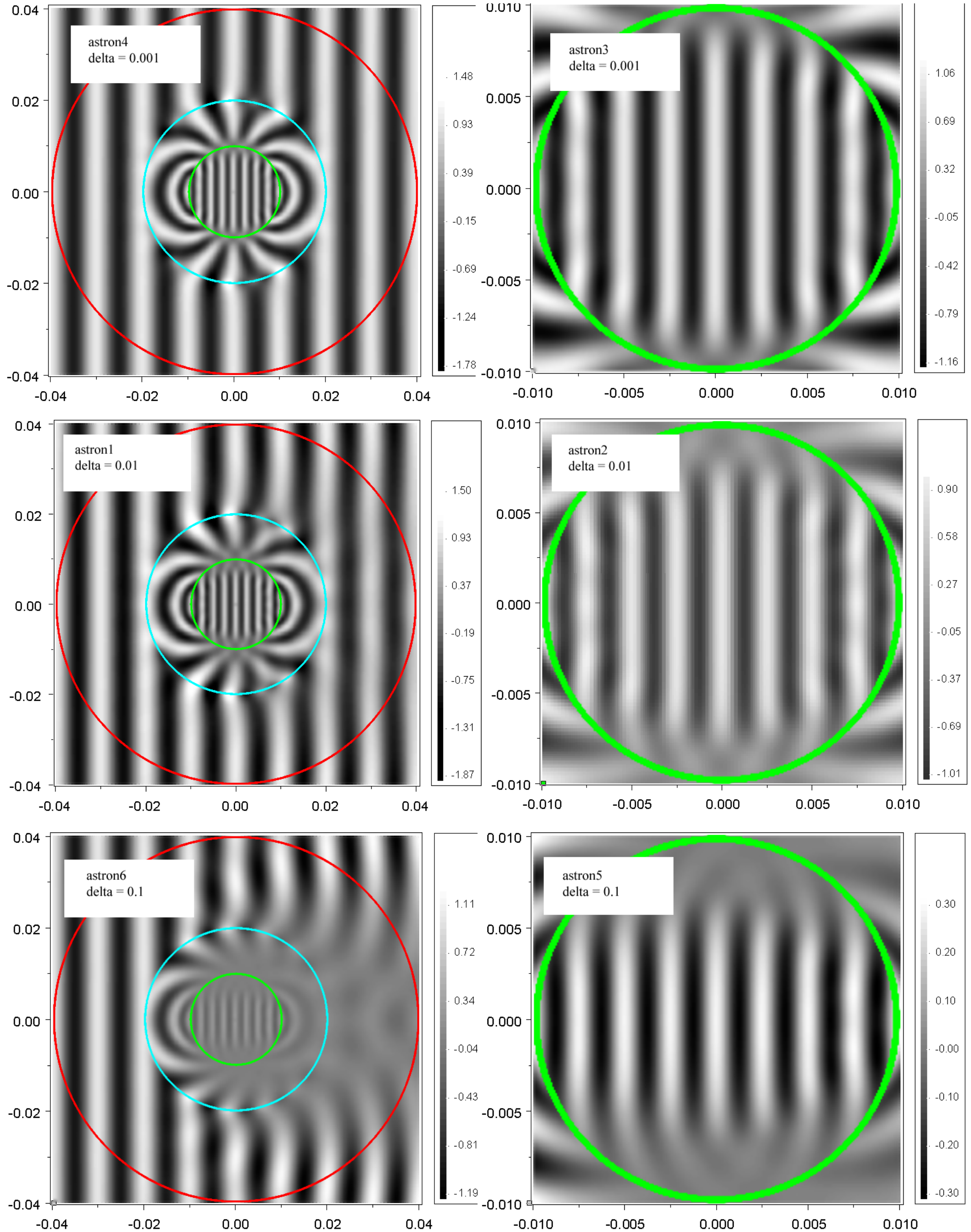


1a)



1b)

An optical turbine. A plane wave entering the red sphere from the left is captured and compressed inside the green sphere. **a)** A ray picture which shows only part of the rays being captured **b)** An exact solution of Maxwell's equations. The green sphere is filled with the compressed contents of the red sphere as predicted. The region outside the blue sphere is free space.



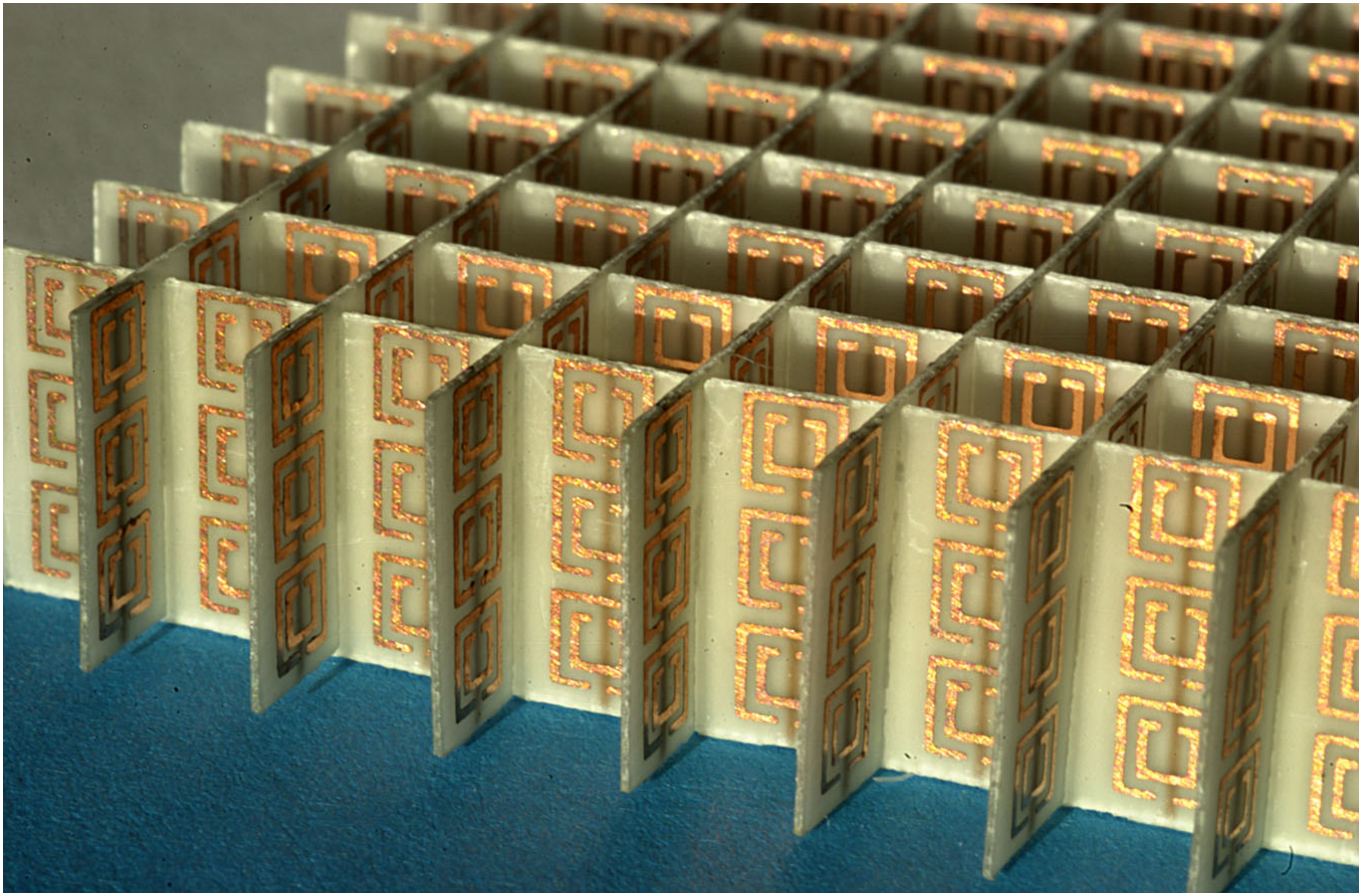
Three separate calculations of H_z for a system containing a negatively refracting material between the cyan and green cylinders, and a high refractive index material inside the green cylinder. On the left is an overview showing all three cylinders: r_1 - red, r_2 - cyan, r_3 green. To the right an expanded scale showing just the inner cylinder.

The top pair is calculated for low losses, $\delta = 0.001$, the next pair for $\delta = 0.01$, and the bottom pair for $\delta = 0.1$.

Note how as the loss is increased the fields are confined to the region occupied by the rays.

Negative Refraction – Metamaterials Issues

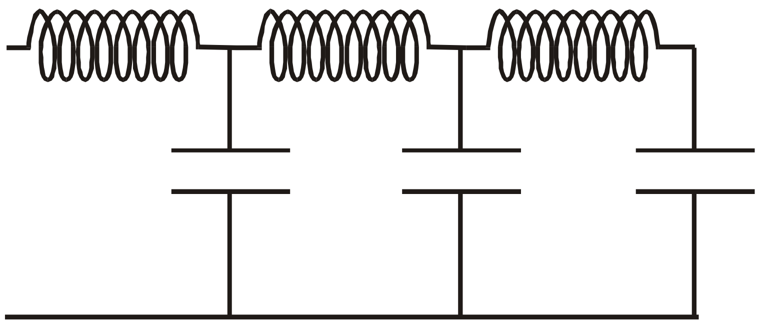
Negative refraction: $\varepsilon < 0, \mu < 0$



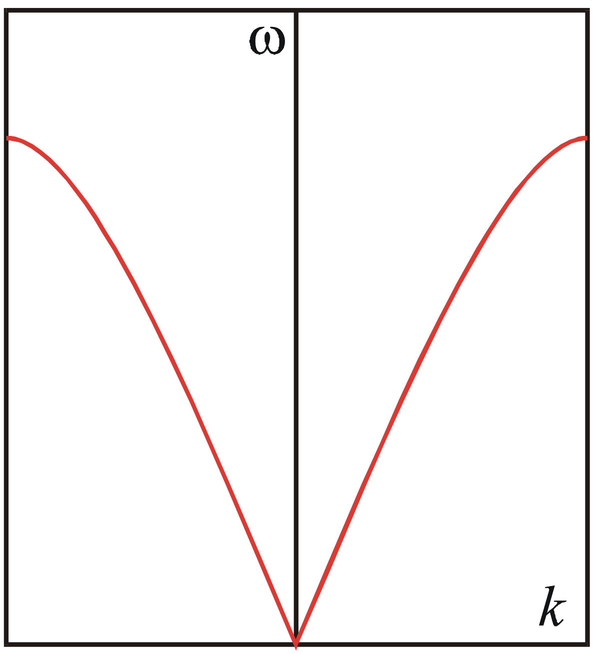
Structure made at UCSD by David Smith

Wave guides as Negative Refractive Index Metamaterials

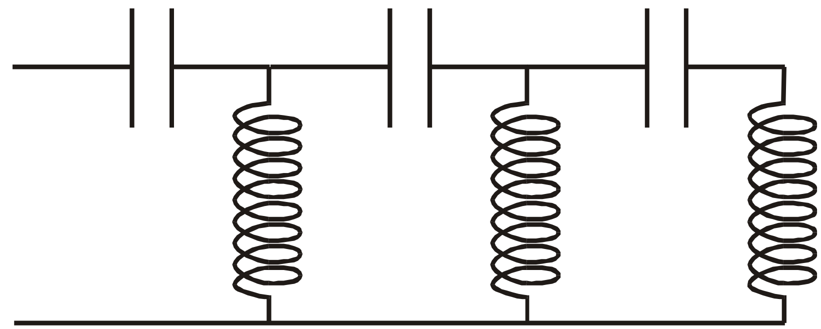
In a conventional wave guide,



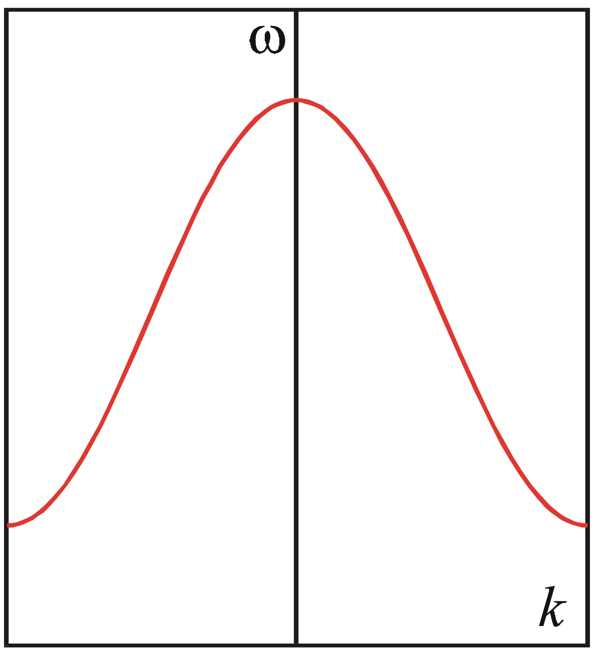
waves travel with a *positive* group velocity characteristic of positive refraction,



In an inverted wave guide:

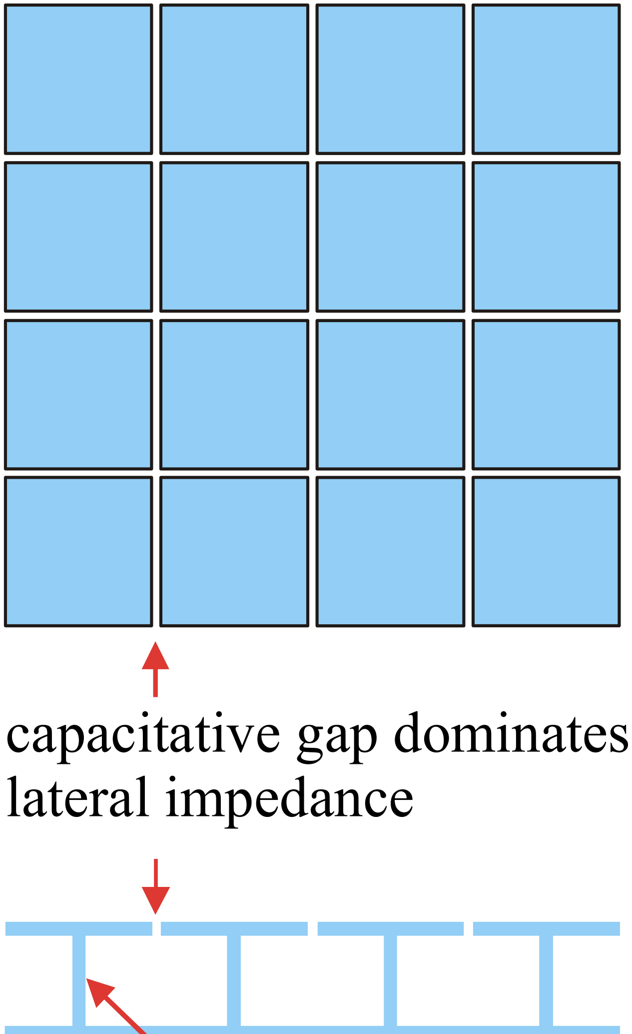


waves travel with a *negative* group velocity characteristic of negative refraction



2D Realisations of NRI Metamaterials

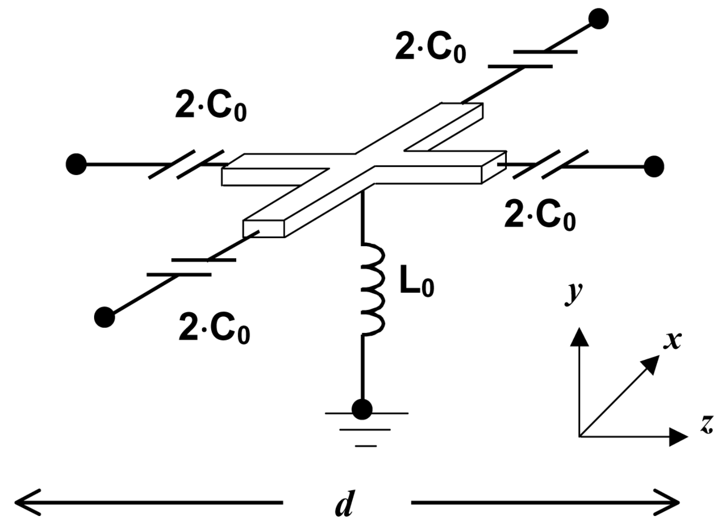
This 2D structure is a NRI material:

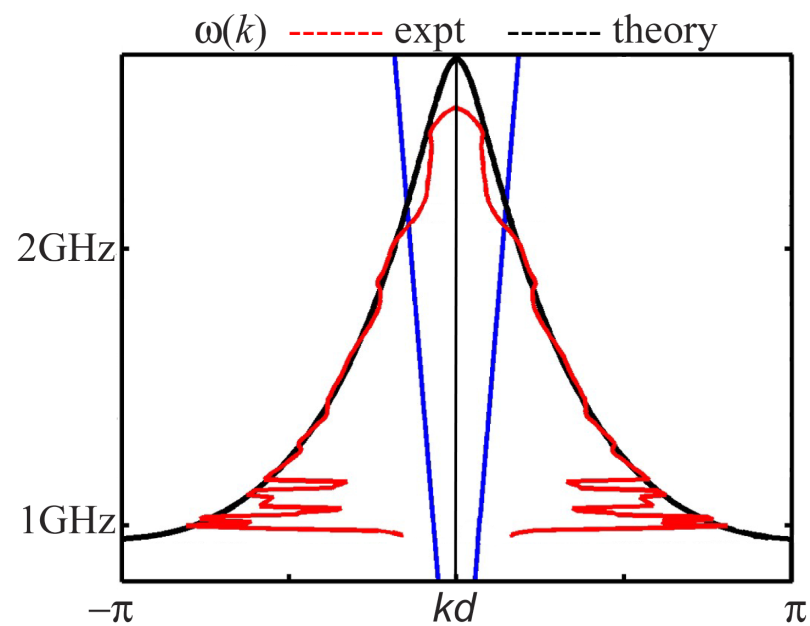


capacitative gap dominates lateral impedance

inductive strut dominates vertical impedance

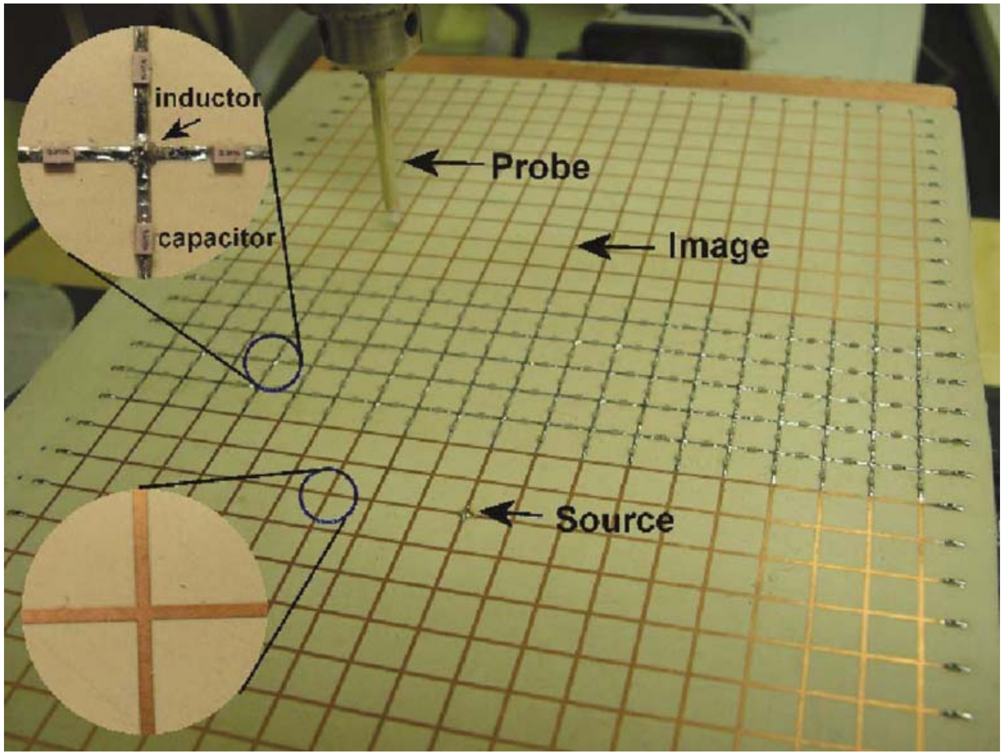
Alternatively we can hard-wire with lumped capacitors and resistors:



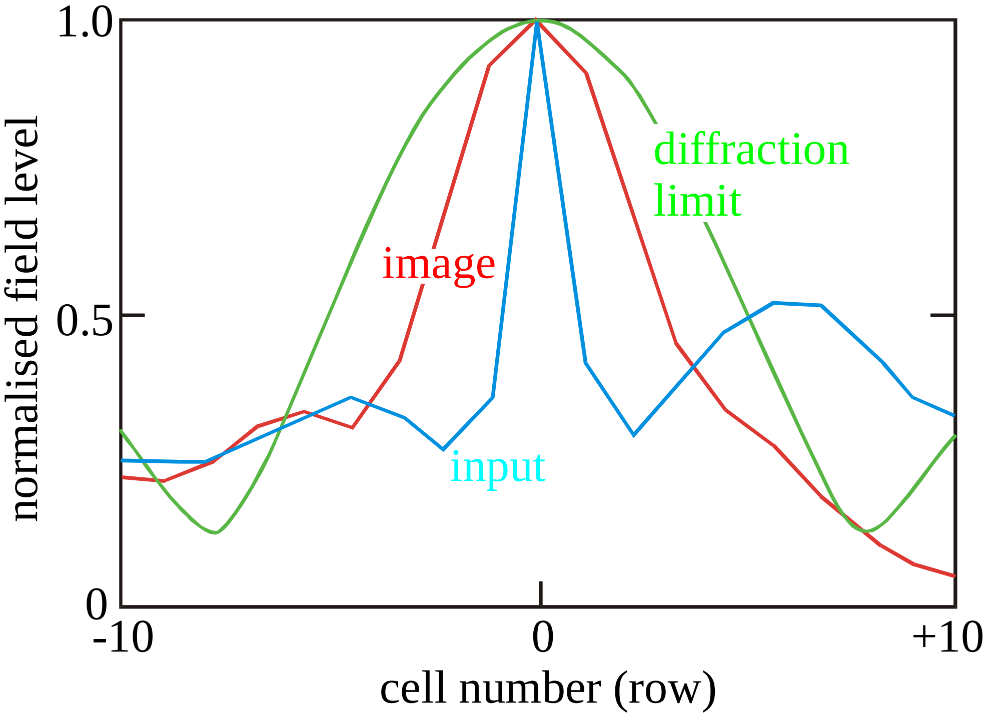


Sub-Wavelength Imaging Using Metamaterials

Grbic & Eleftheriades: PRL **92** 117403 (2004)



The left-handed planar transmission-line lens. The unit cell of the left- handed (loaded) grid is shown in the top inset, while the unit cell of the positive-refractive-index (unloaded) grid is shown in the bottom inset.

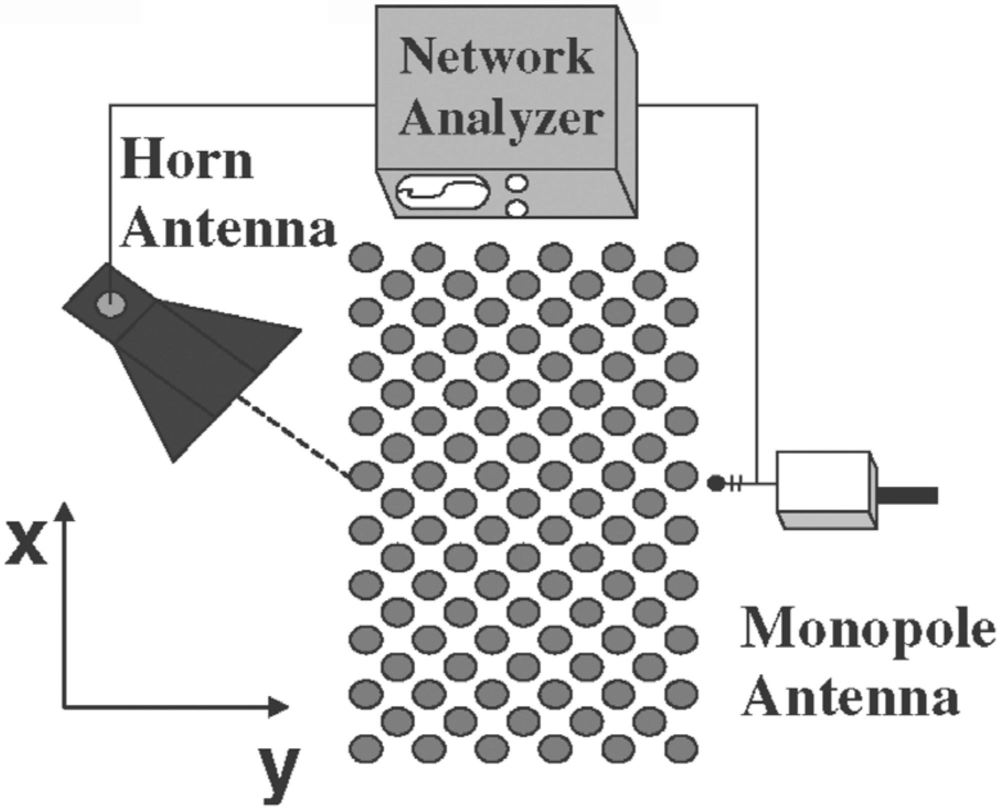


The measured vertical electric field at the source (blue curve) and the image (red curve) planes along with the theoretical diffraction-limited (green curve).

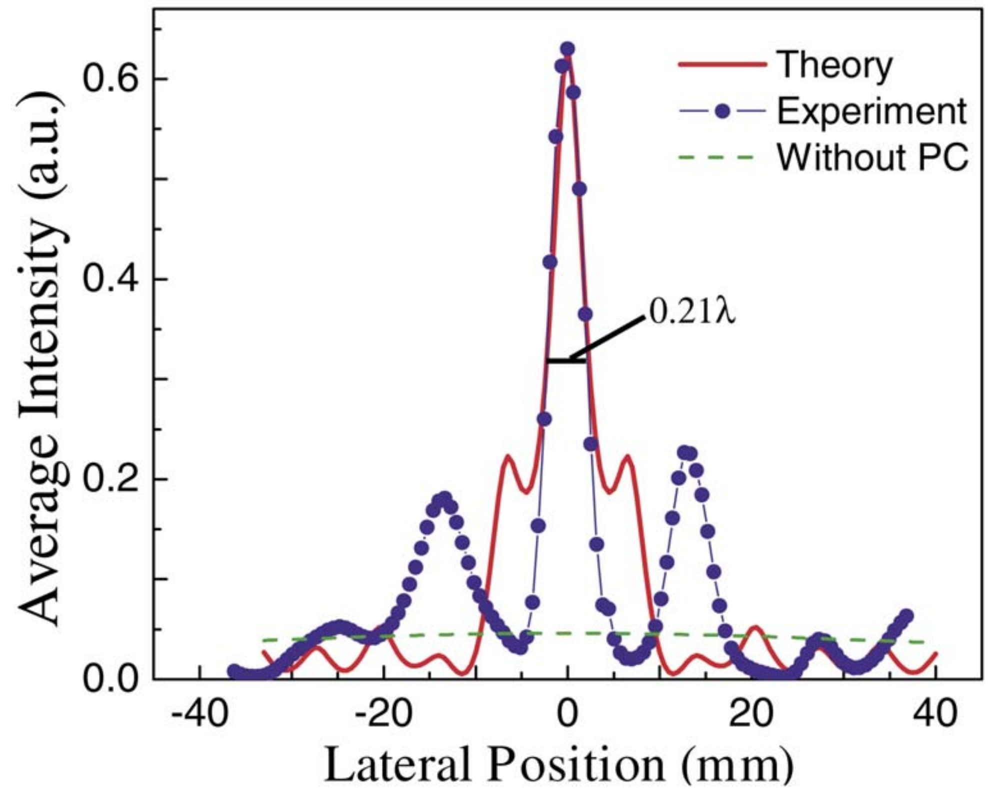
Sub-Wavelength Imaging Using Photonic Crystals

E. Cubukcu, K. Aydin, E. Ozbay, S. Foteinopolou and C.M. Soukoulis

PRL **91** 207401 (2003)

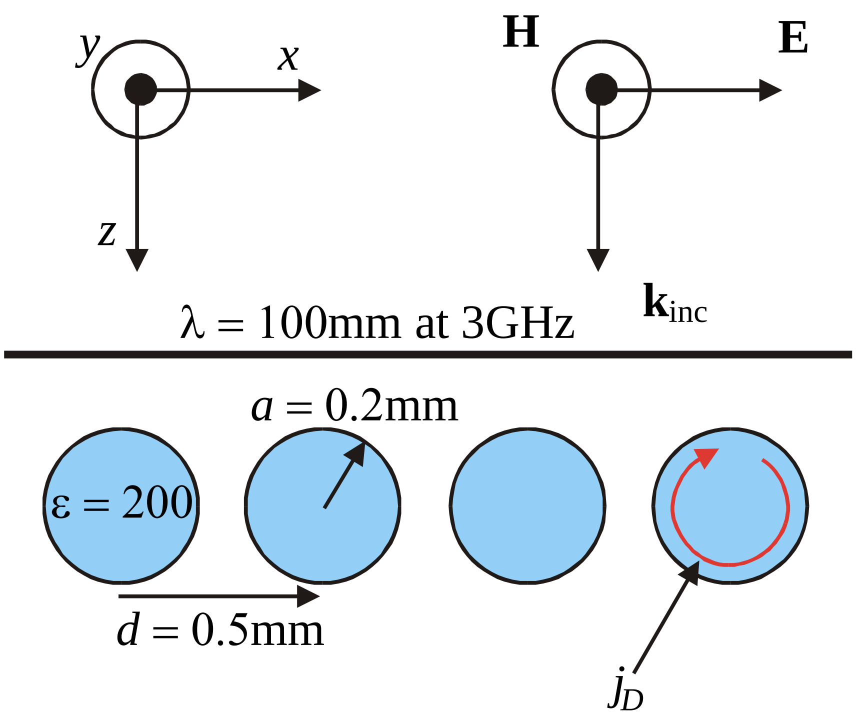


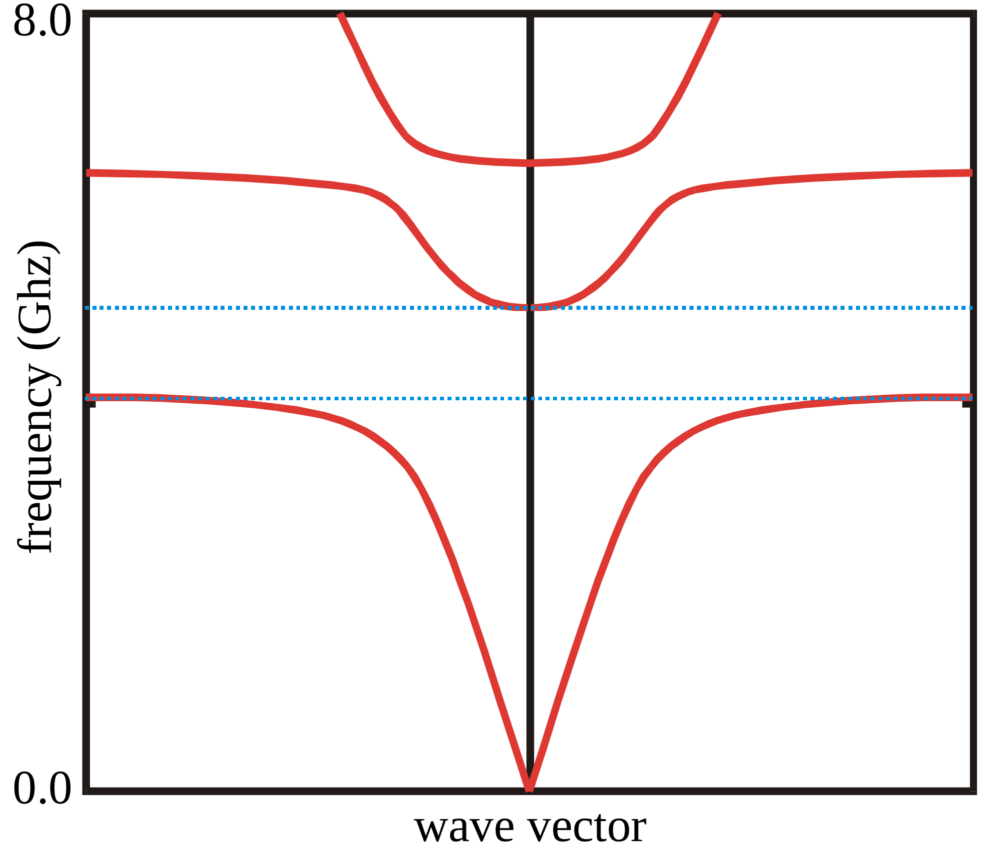
Experimental setup: a horn antenna is used as the transmitter, and a monopole antenna is used for focusing measurements. For both cases a monopole antenna is used as a receiver.



Blue: measured power distribution at image plane
Red: calculated power distribution at image plane
Green: power distribution at image plane without the photonic crystal.
Full width at half maximum of the measured image is 0.21λ.

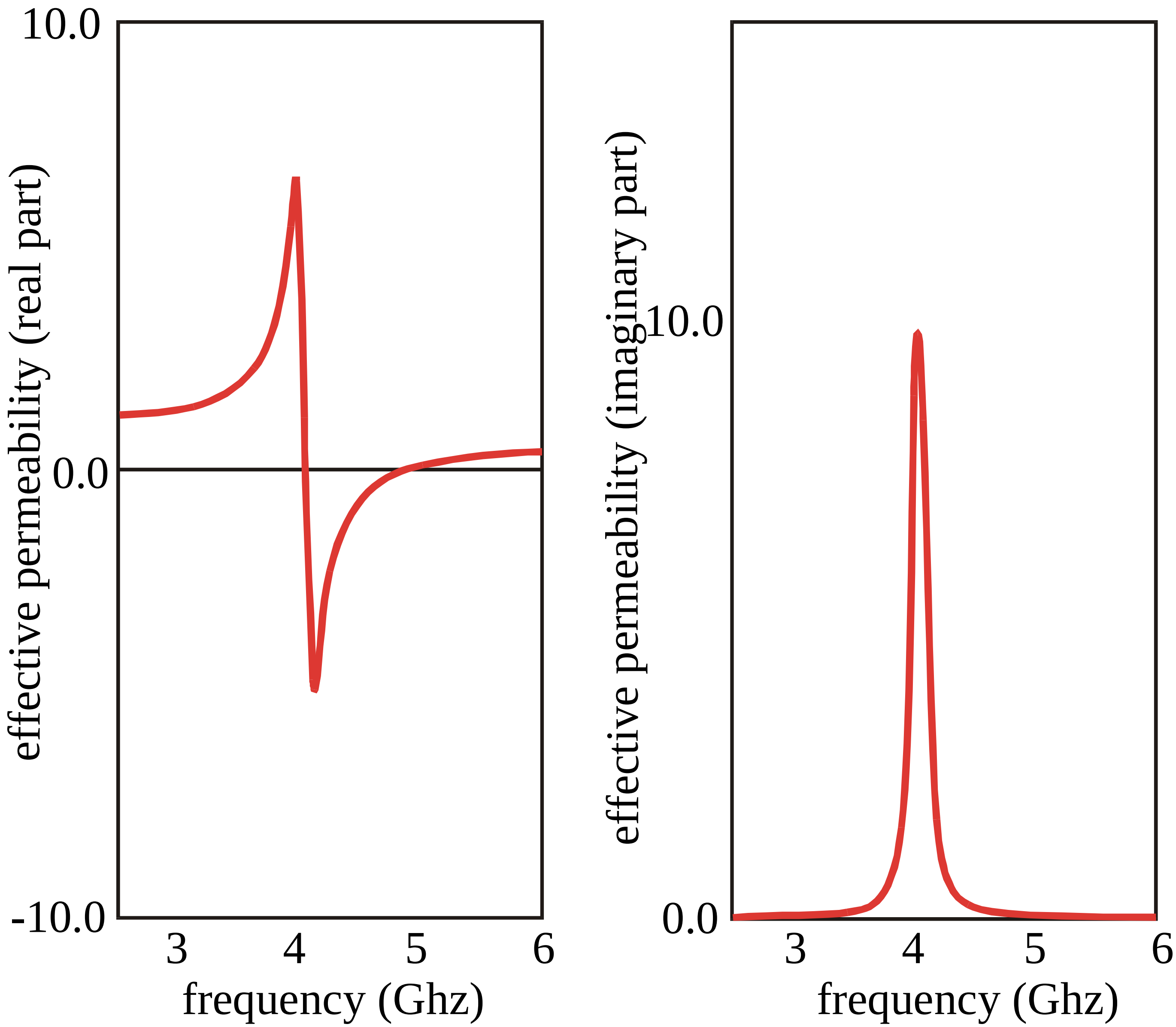
The Dielectric Route to Negative μ: Mie Resonant Cylinders





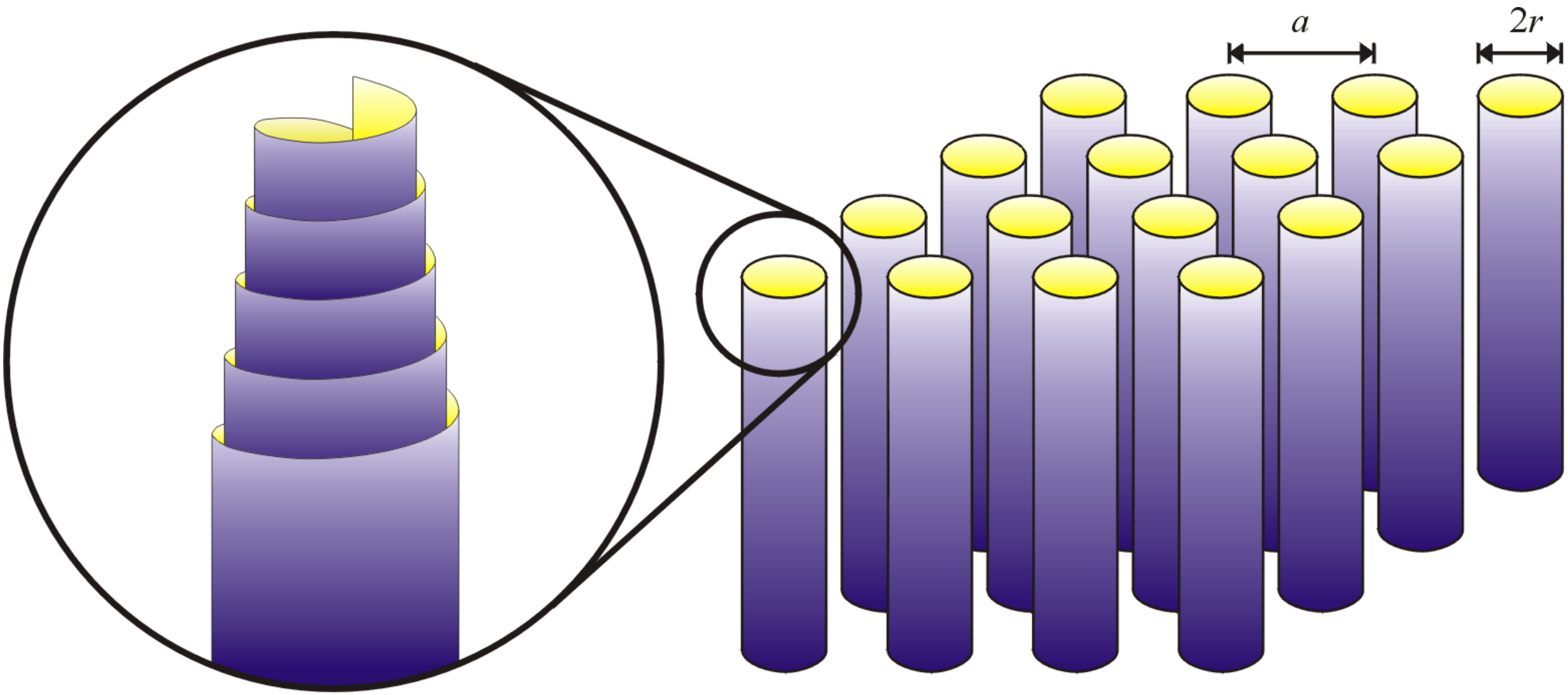
The Dielectric Route to Negative μ

Permeability of an array of Mie-resonant cylinders: $\varepsilon = 200 + 5i$



A metamaterial with μ < 0 at 21MHz

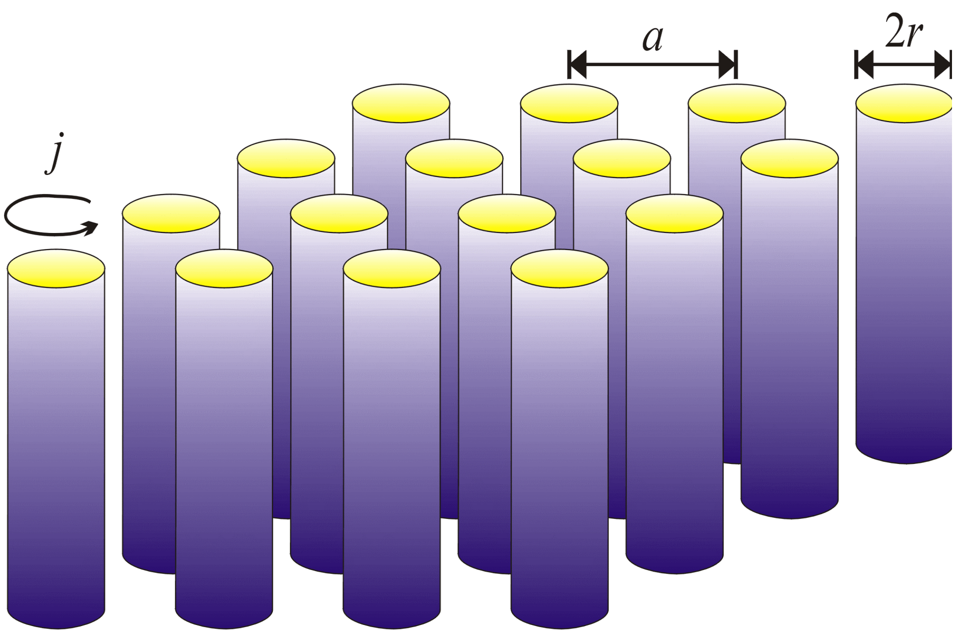
The ‘Swiss roll’ structure comprises rolls of insulated copper sheets. The rolls are typically around 1cm in diameter and resonate around 21MHz. The circulating currents give a magnetic response.

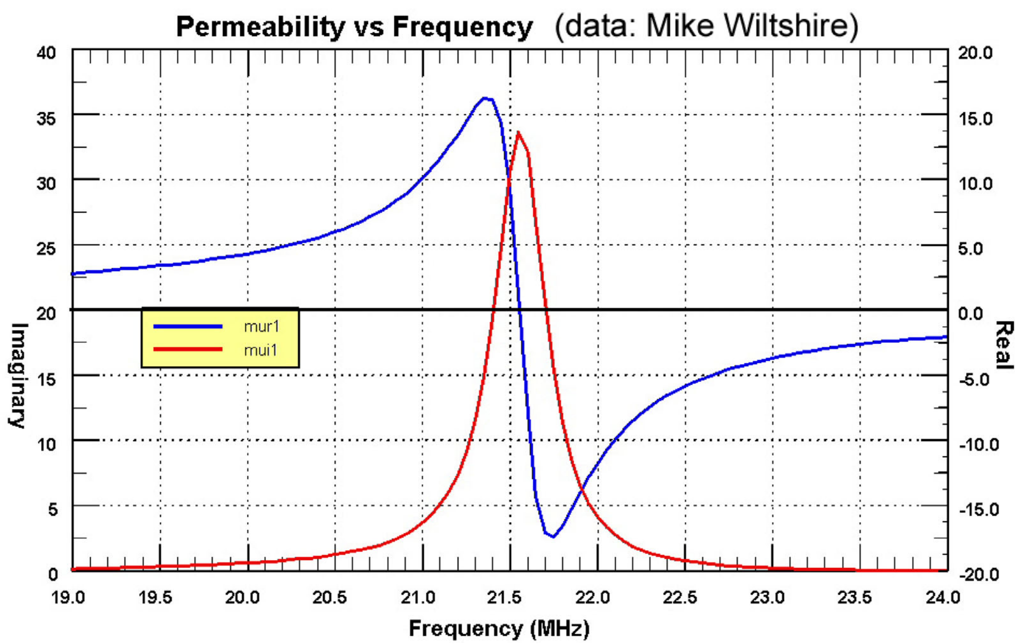


Magnetic Wires: Endoscopes for the Magnetic Field

At resonance each 'Swiss roll' behaves like a magnetic conductor, and is a conduit for magnetic flux. An array of rolls behaves like an optical fibre bundle, each roll capturing magnetic flux at the resonant frequency and transporting it to the far end. There is zero response to a quasi static field. Resolution is limited by:

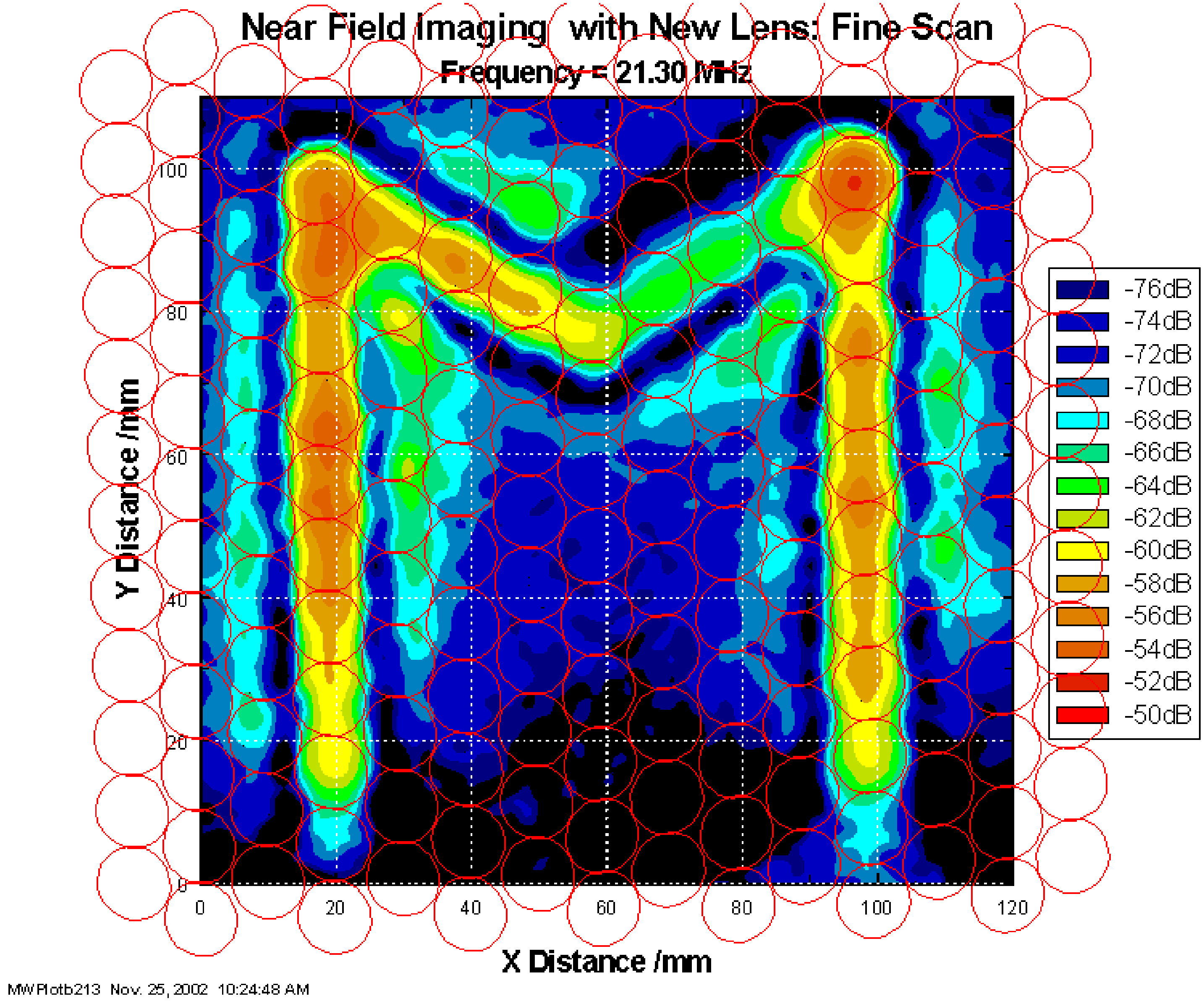
- losses in the rolls
- the spacing between the rolls





Near field imaging with the new lens

data: Mike Wiltshire & David Edwards



Negative Refraction
Designing Our Way Out of Trouble

Negative Refraction
The Near Field Approximation

When all dimensions in the problem $<< \lambda$, then:

$$k_z = i\sqrt{k_{||}^2 - k_0^2} \approx ik_{||}$$

the **E** fields dominate P-polarised light, and

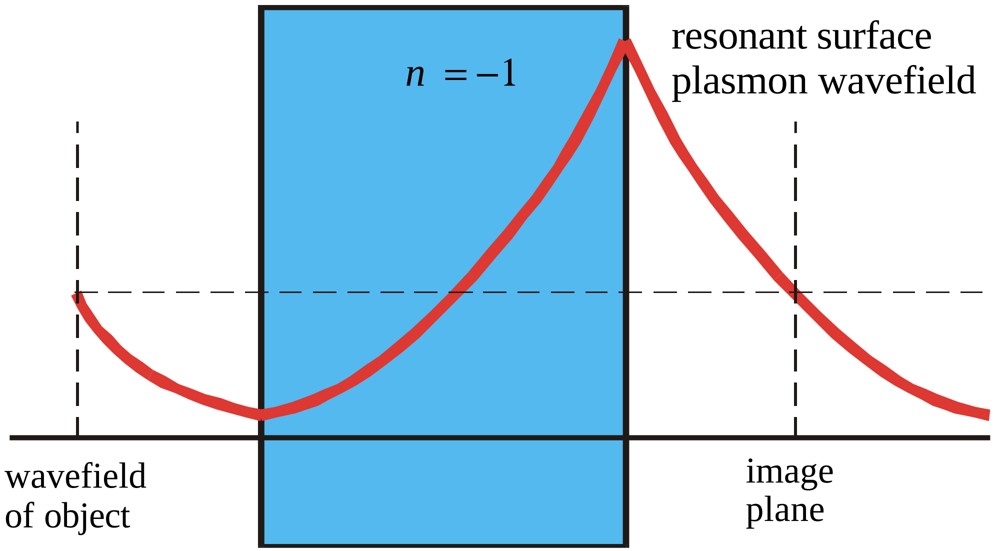
the **H** fields dominate the S-polarised light.

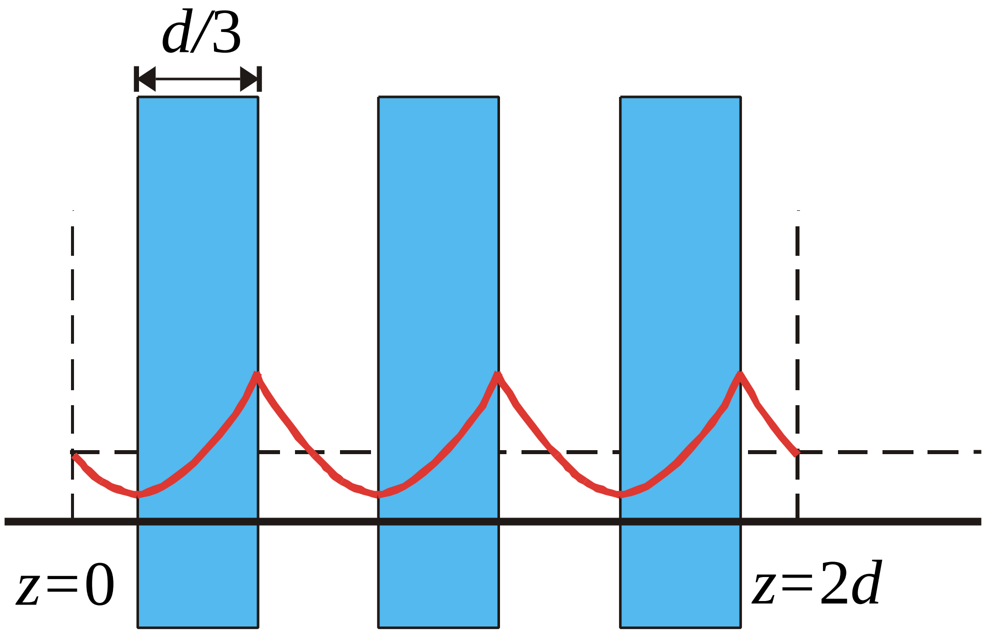
Stay with the P-polarised light and we need only fix $\varepsilon < 0$ as μ does not matter.

There is great potential for sub wavelength focussing by metallic slabs in the optical region.

Optimising Performance: the Layered Lens (1)

Absorption is a problem because of losses in the surface plasmon resonance. Cutting the lens into several mini lenses* reduces the maximum amplitude of the wave field and hence cuts the losses which in turn enhances the resolution.





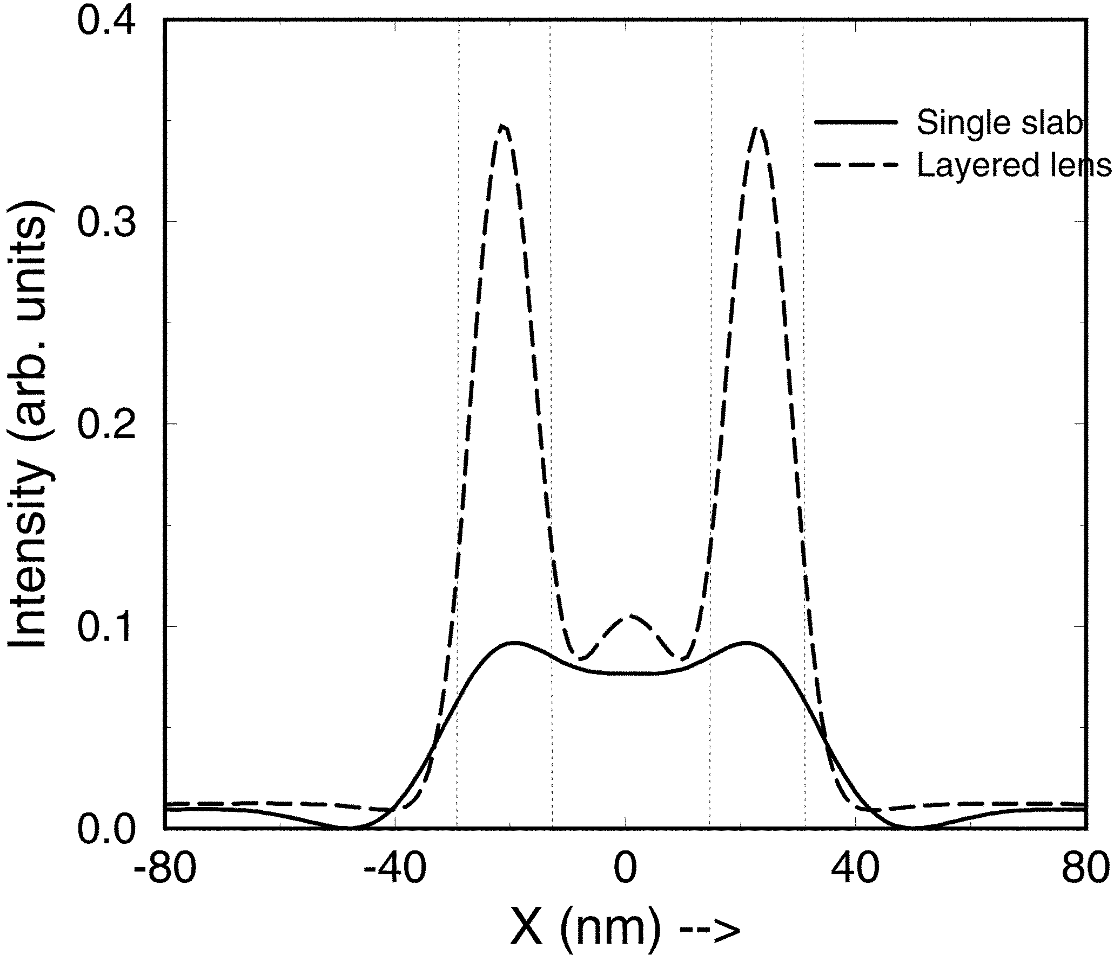
* see also:
E. Shamonina, V.A. Kalinin, K.H. Ringhofer & L. Solymar, *Electron. Lett.* **37** 1243 (2001)

Optimising Performance: the Layered Lens (2)

Reduced losses in the layered lens leads to enhanced resolution. The object comprises two slits of 5nm width and a peak-to-peak separation of 45 nm.

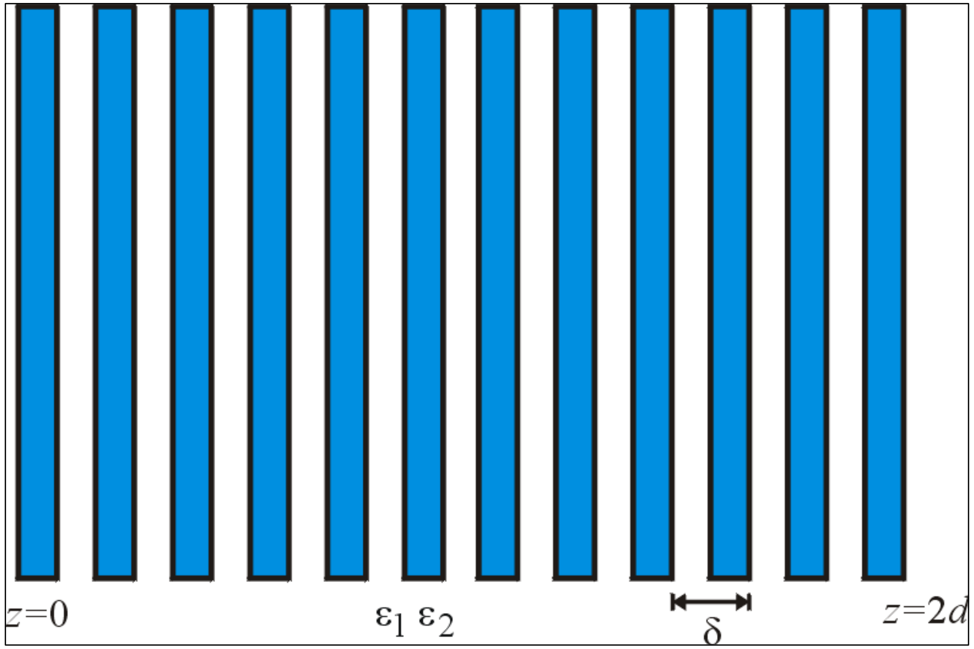
dashed curve: single slab of silver, $\varepsilon = -1 + 0.4i$, of thickness 40nm

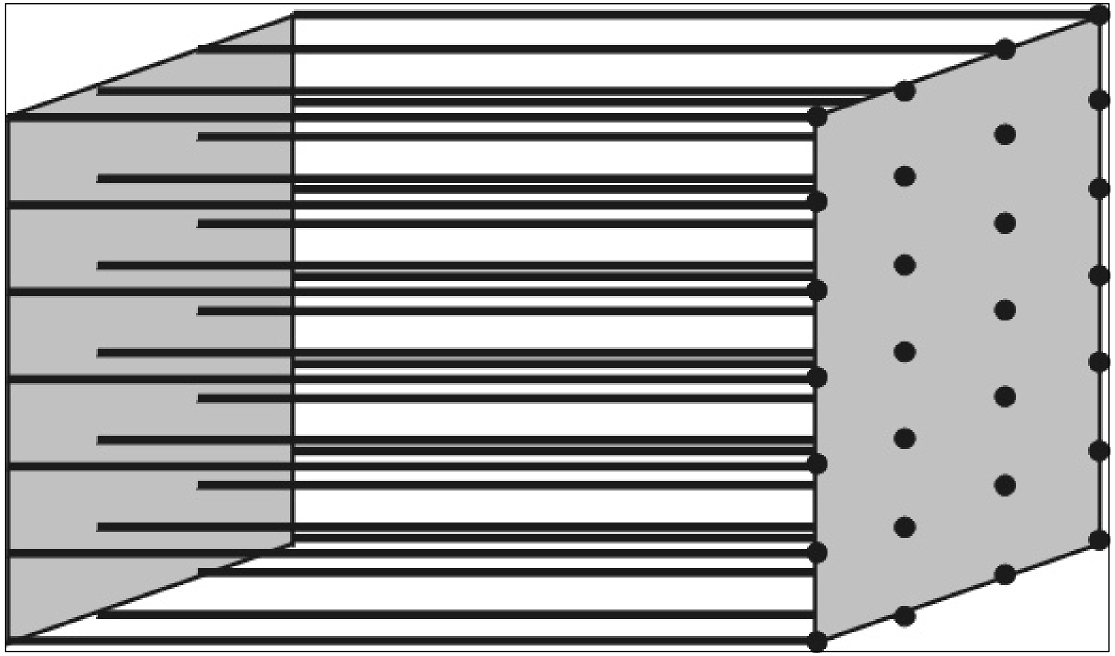
full curve: layered stack comprising 8x5nm of silver (i.e. same total thickness).



The Layered Lens = Near Field Optic Fibre

In the limit that the lens comprises many thin slices and $\varepsilon_1 = +1$, $\varepsilon_2 = -1$, a layered medium is effectively a fibre optic bundle with the unique capacity of guiding the near field. Electrical objects placed on one side of the layers are transmitted undistorted to the other side. The two sides are 'hard wired' together.





The Challenge of Negative Refraction

JB Pendry
The Blackett Laboratory, Imperial College London
http://www.cmth.ph.ic.ac.uk/photonics/

Recent Reviews of Negative Refraction

Negative Refraction
Contemporary Physics **45** 191-202 (2004)
JB Pendry

Reversing Light with Negative Refraction
Physics Today **57** [6] 37-43 (June 2004)
JB Pendry and DR Smith

Metamaterials and Negative Refractive Index
Science **305** 788-92 (2004)
DR Smith, JB Pendry, MCK Wiltshire